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Journal
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MAN IN AN ENGINEERING WORLD.*

BY

DUGALD C. JACKSON, D.Eng.,

Past-president of American Institute of Consulting Engineers,
Past-president of American Institute of Electrical Engineers.

It is an honor to lecture in this grand building of the distinguished and influential Institute which was established as a memorial to Benjamin Franklin, one of America's greatest men. Professor Edward Mead Earle has referred to Franklin as "the patriarch Benjamin Franklin, whose fame in Europe and America was probably greater than that of any other personality of his age." His biographer Albert Henry Smyth says "Across the entire eighteenth century his shadow falls, and all its paths are haunted by his presence"; and again, "There is probably no parallel in the history of literature to the publication of works of such variety, value and enduring fame in such modest and unpretentious manner." The liveliness and scope of his world-wide fame is further illustrated by Honoré Balzac's humorous remark that Franklin invented the lightning rod, the hoax and the republic.

FRANKLIN'S ATTITUDE.

We Americans all think of him as a great diplomat and a great discoverer in physical science, but we tend to overlook another side of his interests in which he also was great, and often ahead of his times. Crowded as were his days with diplomacy and business throughout the years of his maturity,

* Presented at the Stated Meeting held Wednesday, March 15, 1939.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

his mind nevertheless was always lively to the phenomena of things around him and he was constantly wishing to make applications of physical science contribute to the convenience of himself and his fellow citizens. Accurate in observation of things around him and full of reflection regarding them ("ideas will string themselves like ropes of onions" he one time remarked), in free moments he turned his mind to scientific discovery and invention. His inventions were in as wide a field as the range of his multi-faceted mind and even invaded engineering. He discovered or enunciated the principles of, and invented, lightning rods, economical heating devices, towing tanks, improved street lamps, watertight compartments for ships, bifocal glasses, a letter copying press and other devices, of which most (like bifocal glasses) still prove a blessing in the world. For a brief time, somewhat against his will, he performed duties as a military engineer building forts on the Indian frontier in western Pennsylvania. He had a genius for organizing people for mutual service.

We revere Franklin as a great and good man; a man respected in his times for his activity of mind and his good common sense; a man using the phrase "honesty is the best policy" and acting on that policy; a man with a mind in lively contact with all around him; a man interested in his own welfare and also interested to aid others for their welfare. These are all fine qualities. Added to these are his unusual powers of observation, his keenness and excellence in physical experiment and power of interpreting the results, his friendly qualities, his emphasis on thrift, his confidence in the serviceability of productivity in agriculture and the arts, and his diplomatic merry ways of dealing with others. His life was mostly lived before the advent of the Watt steam engine and the factory system which so startlingly altered the world of engineering.

What of such a man in this century? Had Franklin been born in 1906 instead of 1706, with all his genius and all his desire to promote the welfare of man, in what streams of activity would he now most likely be found? That is an interesting speculation for which no positive answer can be formulated, because Franklin's mind was not exposed to any full equivalent of the conditions of this actively industrial age.

One can, however, arrive at a pretty keen estimate of his reactions by considering each of his actual episodes and his reactions thereto and then reflecting on the modifications of his activities that his mental qualities would impose if exposed to episodes modified according to our days.

Reflection along these threads convinces me that, in our days, Franklin would have become an investigator in physical science, a notable inventor, and a great industrial leader, all stimulated by his lively curiosity regarding occurrences and their causes and his strong personal wish to improve the conditions of mankind. Moreover, he would have kept a lively eye turned upon the attitude of government toward independence of industry and a diplomatic guiding hand in the situation. These roads would be the roads of fascination for him. The development of industry could attract his mind in our day as did the development of the colonies in his day, and he would become absorbed in the development of industry because of a conviction that advantage to his fellow citizens would come along that route, just as he was convinced that advantage to his fellow citizens in the colonial days would come through developing colonial solidarity and productiveness. Franklin was an ardent proponent of experimental investigation such as we now call scientific research, and of the usefulness to mankind of making prompt applications of newly recognized natural phenomena to contribute to man's safety and convenience.

A reverse mental attitude was exhibited when eminent Englishmen some years ago vocalized the opinion that a moratorium on scientific research should be declared in effect until the world caught up with what it had. It was somewhat difficult to decide whether this vocal opinion was in earnest or only expressed characteristic witticisms to be taken catch-as-catch-can. You may remember that an Englishman also said that "A good joke is the one ultimate and sacred thing which cannot be criticized." However, a good part of the English-speaking world-citizenry seemed momentarily to take in earnest the opinion regarding the desirability of a moratorium in scientific research; and when Dean W. B. Donham (banker, educator, leader of the Graduate School of Business Administration in Harvard University) several years

ago expressed an opinion that, "The impact of material progress has lessened, destroyed or prevented the development of social customs and control, which alone enable mankind to live as a social animal," he was in full earnest.

Such proposals repeated again and again by many men several years ago are a serious challenge to thought, thrown before Man enmeshed in the affairs of an engineering world. Are such proposals right or are they wrong? Was Franklin's attitude toward the serviceability of further knowledge and its prompt application to useful purposes only apt for conditions when research and industry were yet lean? Is it desirable to restrain scientific discovery and engineering invention for awhile and turn our major efforts toward modifying human attitudes in other fields of activity for the purpose of preventing misuse of engineering invention? Or should we continue to encourage the active prosecution of the scientific and engineering occupations while we also endeavor to make ameliorating adjustments in all social relations?

I confess to affirmative support of the last stated thought. I believe that this is the view that Franklin would have adopted. But I will try to lay before you open-mindedly the grounds for reaching an opinion. I am referring here to circumstances relating to engineering. If we add preventive medicine to the case, the arguments are similar but more powerful, because preventive medicine may be given first place in importance to the welfare and safety of modern peoples, and engineering is a close second. Indeed, preventive medicine leans in some things directly on engineering, as in sanitation, and the two go hand-in-hand. It is also appropriate here to say that no intelligent being should oppose regulations intended solely to prevent misuse or injurious use of scientific discoveries and inventions, whether in engineering affairs or medicine. In standing favorable to such regulations one assents to the social fact that one has no rights which are not associated with coördinate duties.

WORLD-WIDE IMPLICATIONS OF LECTURE TOPIC.

My topic of Man in an Engineering World is world-wide in implications, but its import now is greater for the western world than for the Orient. The import rests with notable

emphasis on our own national surroundings and I will use our own relations for primary material, but draw upon other parts of the globe and also former times in history for supporting illustrations and for illustrative contrasts. As engineering is a primary word in the phrasing of my topic, I will here bring to your mind the definition of engineering.

Engineering is usually defined as the art of directing the forces of nature for the service of man. It is a process of planning, organizing and executing work which comes within the foregoing usual definition, and is distinguished from artisanship by its demand on the intellectual qualities to bring together and combine physical elements known to science or artisanship into a coördinated unity (such as a structure or process or machine) for the convenience of man. Engineering is a profession which requires highly developed inductive intellectual qualities. It is a scientific profession, but has great influence on political economy. It rests on observation, scientific discovery, and invention; of which three processes, the last is of synthesis and draws partially on deductive intellectual qualities. The qualities of invention are constantly utilized by engineers, although inventors need not always be engineers. The processes of scientists also are constantly utilized by engineers, although scientists need not be engineers. The distinction between the last two groups of men lies in tastes and individual interests.

Engineering influences germinated in prehistory and grew throughout historical time. They have blossomed most vigorously during the last two or three centuries and will continue to blossom in the duration of time while man's mental characteristics continue of their presently established qualities. Engineering processes possess potentialities for both good and evil. We here consider both potentialities and balance them against each other, but cannot in a single lecture exhaust this discussion in its details. The best minded scientific research men and the best minded engineers

> "Go forth to seek: . . .
> The skyline is a promise, not a bound."

Along with this goes a recognition of appropriateness in the development of a sense of brotherhood and mutual helpfulness

among men. Many others of the human kind join this group of the mutually helpful. Others among men hold a quite different mental attitude. And some even deny the propriety of extending mutual helpfulness generally among men.

MUTUALITY OF INTEREST LEADS TO DEMOCRACY.

At this point I will remind you that history is man-made, and that the fine-spun arguments of some people are as tenuous and as incompetent to survive against determined attack as the house built by a spider; and, truly, the frailest of all houses is a spider-spun house. Like spider webs, most of the "isms" that come into the world are doomed to ultimately disappear because they cannot survive logical attack from intelligent minds; but unhappily, because of their intolerance, they often are the causes of much suffering while they exist. Candid scrutiny of the stream of history shows that no historical event among human beings is inevitable, but every event is a consequence of human attitudes. Men, like seeds, do not all come to equally desirable fruits and flowers. Untoward events arise out of human errors, angers, jealousies, vanities, selfishnesses, and greeds; while fortunate events arise out of the developed sense of mutuality of interest among individuals and nations, which sense leads to platforms of generous fair dealing.

From the latter, when fully established, arises democracy; but as Dr. George Norlin has said, "Democracy is not an accomplished thing, not an established thing. Rather it is a dynamic faith which has always had to do battle. Just now it has its back to the wall abroad, and we ourselves honor it as much in the breach as in the observance. . . . To say that democracy does not work, and that there isn't anything that we can do about it, is simply to say that we are too indifferent, too irresolute, too spiritless to do anything about it." The indifference and irresolution which President Norlin refers to are characteristics of large parts of every population, whose ungraceful attitude of lacking self-reliance is gracefully mirrored by a quatrain of Omar Khayyam:

> "For in and out, above, about, below,
> 'Tis nothing but a magic candle-show,
> Play'd in a box whose candle is the sun,
> Round which we phantom figures come and go."

Here let us observe that democracy arises out of civilization, which rests on the growth of mutuality in interest of populations living in intimate community relations, or otherwise with intimate contacts amongst each other. Intimate community life and intimacy between communities are made practicable by engineering works and the development of civilization is inseparably related to the growth of engineering. I can here only outline the fact to you, but any of you who are interested to get a fuller view of these inseparable relations may refer to a small book entitled "Engineering's Part in the Development of Civilization" which was recently issued by the American Society of Mechanical Engineers as a reprint of a series of six lectures which I had delivered on the subject.

Incipient civilization, with its engineered villages and other fruits of crude engineering, began in prehistoric times. It has broadened, deepened and sweetened during the ages of history while engineering experience and engineering practices have contributed widened and improved contacts between populations. During this broadening, deepening and sweetening through the ages, civilization has taken on aspects of change, which have profoundly affected the living conditions of men and the relations of men with each other. The conditions and relations continue to change, because scientific discovery and invention constantly bring new light and new practices into engineering. Consequently we live immersed in an æther of change. We live more securely and comfortably today than ever before, but by no means have reached an ultimately satisfactory level. Certain of our habits as moderns are vestiges of ancient taboos. We have something of the totemist attitude left in us but generally-spread education regarding the phenomena of nature has eradicated, in this country, the influence of animism, taboos and the like from minds of the educated.

Our attitudes (among educated people) toward mascots, taboos and the like are as a rule recreational or are humorous mannerisms. We therefore live lives which are less impressed by fear and suspicion than our primitive ancestors on whom taboos weighed heavily, if analogy with the ways of primitive men existing today is sound in application to our primitive

ancestors. Even a thirteenth story in a hotel now holds terrors for few of us, and similar superstitious terrors which in multitudes oppressed the primitives, are without force with us. What agencies have brought to pass these changes? They are civilization, education, knowledge of facts, contacts of peoples with peoples—for the major features. Engineering works which make civilization possible by making community life practicable, and provide ready communication between communities, have a credit in the situation. For illustration, engineering made possible that thirteenth story which might be taboo in the hotel building, but the education which robs the totem of its sacredness and the taboo of its force comes out of community education made possible by engineering.

We, as a consequence of our mutual experience, our habits of experiment, our education, and our mental vision of relationships are competent to think in abstractions and we smile over the taboos. Primitive man could not compass abstract thoughts and his taboos therefore were very real to him when once established. So, in this engineering world we are absolved from many terrors which weighed heavily on our primitive ancestors. We also, through the results of our higher order of engineering, enjoy security, comfort and convenience of which they were not competent to even dream. Superstition still hangs heavy in those parts of the world where illiteracy is rampant and communication by engineering means is slight. Mystical qualities are observed by the inhabitants in such areas for a variety of objects which when subjected to our exact and objective observation appear to us of only ordinary character.

EFFECTS OF ENGINEERING IN EARLY HISTORY.

In the early days of history, that is in the period soon after permanently inscribed records came to be made, the Egyptians, Elamites, Akad-Sumerians, Babylonians and others living in rather dense populations produced quite notable cultural establishments, which are often referred to as civilizations. These peoples planned and executed important engineering works. Their rulers, nobles and priests lived lavish lives, but the great proportion of the populations

were commoners or slaves who lived miserably. Cruel superstitions rested as a cloak over all. The contacts between different nations were too tenuous to allay suspicion and enmity, engineering being not yet competent to provide ready means of travel or quick means of intercommunicating. When war arose, an invasion commonly spelled the sword, famine or slavery for the population of the invaded nation, and war was a commonplace of life in those days.

The engineering works of these peoples in the way of irrigation projects and masonry or brick structures arouse present day admiration but the productivity of the peoples was sufficient to support only a few in comfort (and these in luxury) while the many were without assurance of continuous adequacy of food or protection from the elements. The source of power for the engineering works was muscle-power of slaves, and slaves were expendable property just as horse flesh was with us before the advent of the automobile, truck and tractor. Indeed, it is my impression that, due to the contrasting higher level of education and of veterinary medicine, the horses on farms of our western plains were under more favorable conditions for maintaining life than the labor-slaves of the Pharaohs. Note that in the days of the Pharaohs engineering had as yet secured but a narrow scope. When we think of those early historical times, we must remember that engineering was directed substantially exclusively toward the planning of cities, the building of palaces and tombs and memorial structures, the planning and building of irrigation projects, mostly for the benefit of the rich and powerful. The performance of heavy work rested almost solely on animal power or the muscles of human beings.

In a later period came the Minoans of the Island of Crete, the Mycenæans of the Grecian mainland and the Phœnicians of the eastern coast of the Mediterranean Sea, all sea-going adventurers but with the Phœnicians much above all the others. With them engineering grew apace. Harbors and ships of their types were built. Commerce grew all along the Mediterranean shores, onto the shores of western Europe and western Africa, through the Red Sea and

easterly along the shores of the Indian Ocean. It has been claimed that the Phœnicians got as far east as India and perhaps farther. They are known to have traded and planted colonies on the shores from end to end of the Mediterranean Sea and to have traded along the western coasts of Europe and Africa. Their engineering experience and the agencies produced by their engineering ability afforded the opportunities for their far-flung trading routes, and promoted the mingling of ideas from many grades of men and also the mingling of the men themselves, thereby causing a strengthening of interest in mutual relations and education and a weakening of superstitions.

The contacts made and maintained by the active trading stimulated mutual interest and broadened the foundations of civilization, all based on the influence of the engineering activities of ship building, navigation, harbor making and city building, guided by the unusually enterprising and resourceful Phœnicians for the purposes of their trading in pottery, metals, glass, dyes, textiles, personal ornaments and other desirable articles of commerce. The Phœnicians are credited with originating an alphabet to meet their commercial needs, which had an influence on the subsequent alphabets of the Greeks and the Romans, and thus on the whole of our Western World. Superstitions softened, observation of natural phenomena became a part of the life of a sea-going and trading nation, civilization was spread abroad. The life of these people was vigorous and involved much warfare, but it was infinitely better for the commoners than life had been in the older nations where engineering was developed in less scope. These people lived in the period when Hesiod wrote that industry makes work go well but a man who puts off his work is always at hand-grips with ruin.

While the Phœnicians were thus spreading civilization and the engineering arts hand-in-hand, along the shores of the Mediterranean and along western Europe and western Africa, waves of uncouth and illiterate people from the northward and eastward beginning about 3400 years ago engulfed the Mycenæans and Minoans, destroyed their cities and settled in their areas. They came to be known as the Greeks. They were relentless warriors, but soon came into trading

relations with the sea-going Phœnicians and cultivated commercial habits for themselves. They developed an alphabet and a language to meet their needs, cultivated the known engineering arts, and some 2400 years ago were the possessors of the Great Grecian culture which is so notable for its literature, its sculpture and its architecture.

SOCIAL RELATIONS OF CIVILIZATION AND ART.

It is usual to speak of the Grecian result as a civilization —indeed, as a great civilization; but it was not a full civilization, for it rested on a selfish and very cruel slave-economy. A more highly developed sense of human mutuality was essential to round this and similar great cultures into full civilization. The people of western cultures have progressed far in that direction during the 2400 years since the days of Pericles. In the western democracies we are now seeking a full civilization (to which we have made a closer approach than the Greeks or any other of our predecessors) in which it is the ideal that every individual is free to reach such attainment (and is encouraged to do so) as his ability and ethically applied exertions warrant, and in which the spread of engineering becomes so sufficient that every individual who is willing to exert himself may be sure of adequate food, raiment and shelter for himself and his family. Here we are at the center of the question whether we can afford to restrain the active prosecution of scientific research and engineering invention and risk losing opportunities to move nearer to that goal of full civilization. The recent experiences of our nation make a terrible indictment of an economics of scarcity and repression such as our sentimental New Dealers supported for a time.

There is an interesting contrast to be referred to at this point, in the relations to society of civilization on the one hand and of such cultural incidents as art, architecture and creative literature on the other hand. The former, with its development in some degree of ethical principles and codes of morals, is a practical working necessity for man in community living but without significance for an isolated individual. Civilization is not a sentiment, but it becomes bound up with sympathetic (and therefore somewhat sentimental)

relations of man to man. Civilization of some degree is not only a practical necessity for man in community living, but the community life becomes more secure and happier as the aspect of sympathy becomes more complete within the zone of practical application without extruding into sickly sentimentality.

The development of art, architecture and creative literature, on the contrary is purely sentimental and a feature of the life of each individual. The influence of these accomplishments is as grateful to the mind of a lone individual of intellectual gifts as to the same mind when the individual is immersed in community living. Their development therefore is a feature of sentiment alone. They contribute graciousness and repose to life and are needed equally for the individual in isolation, the lone family, and the units of the community.

Life can be secure and happy in an engineering civilization which possesses instruments, utensils, furnishings, building construction, machinery and writing which are of only the simplest substantial construction; but adding the farther touches of practical sentiment through art, architecture and literature gives a desirable touch of softness and sweetness to living. The Greeks attained this for a few among their people. Their engineering was inadequate to enable them to produce sufficient support to all the population in ampleness of food, clothing and shelter so that all might visualize a life of comfort. Their conditions of life ultimately brought their downfall and they came under the domination of the Romans.

EXPANSION OF CIVILIZATION BY ROMAN ENGINEERING.

Road building had developed rather early into an engineering art in the neighborhoods of considerable populations; but for general road building in any extensive country the world found it practicable to wait on the advent of the Roman general, Julius Cæsar, some 2000 years ago. The Roman Empire spread, exhibited engineering skill greater than its predecessors, built notable roads, bridges, water supply structures, sewers, and massive buildings over Western Europe, northern Africa and western Asia, and also built ports for Roman shipping. Conquest and wealth apparently

were the primary objectives of Roman ambition. The Romans pushed their relations in all of their wide area by the sword. But ambition is a nervous thing which often leads to unanticipated consequences, and the Roman occupations wrought out the engineering foundations for great cities in Spain, France, England and elsewhere, besides those in the native land of Italy. Great life was breathed into the communications between the various peoples held together in the Empire, and active commerce ensued. In spite of the wars and the military duties, the lives of the common people were easier than in earlier nations. The development of engineering had resulted in increased production and improved distribution of goods needed for security and comfort in living.

The influence of the Roman Empire carried commerce and the Roman alphabet (over permanently built roads by land and by ships at sea) into wide areas of the earth. This dissemination of a uniform alphabet among widely separated peoples was a magnificent contribution to increasing the sense of mutuality of diverse populations. Unhappily, the Romans, as they withdrew from their far-flung outlying Empire, did not leave behind them a legacy of uniform language spoken by the various peoples. Uniformity of thinking, and affection for the same objects and causes, make tremendous contributions to mutuality of interest among peoples and thus to civilization, and this dissemination of a uniform alphabet over a considerable part of Europe, which the Romans accomplished because of their engineering genius in building transportation-ways and permanent works, was only second in effectiveness to what might have arisen from uniformity of language. The attitudes of those coteries of people and those nations which are now endeavoring to develop independent languages are to be deplored. They are deliberately sacrificing advantages in mutuality of interest which now are to be obtained through the engineering agencies of transportation of people and goods and the quick communication of intelligence; and thus are sacrificed foremost means for cultivating mutual goodwill and mutual support between nations.

It is proper to here emphasize that true civilization grows as a consequence of community contacts among human beings, and intimate community contacts (broadly speaking)

are made possible by and are dependent on the fruits of engineering. Such fruits became vaguely manifest many thousands of years ago and have steadily ripened and increased in scope and variety through those thousands of years, down to the present time. The increased responsibility for mutual sympathy and mutual welfare laid on individuals and on communities by the doctrines of the Christian religion have increased for engineering its opportunities to serve and its responsibilities.

EFFECTS OF ENGINEERING IN PRESENT AGE.

In our present state in the western world we have reached an age when the effects of engineering run through all conditions of living. We of the western world have so profited by active prosecution of scientific discovery, invention and refinement of engineering processes that our conditions of life for all but the most wealthy are in strong (indeed startling) contrast with the woeful conditions of living available for the poor and the moderately well-to-do in those eastern countries which have failed to participate in such developments. We have further improvements to make. For illustration, it is a fact that we have not yet developed the human contacts, in those of our industrial organizations in which production by machines is prominent, so as to reproduce the friendly responses that are possible among industrial groups, and which were observable in the best of old-time hand-work conditions. Such defects, however, gradually become rectified as the modified social relations gain recognition.

With us, all engineering works—good structures for shelter, hard-surfaced roads, bridges, railroads, ships, water control, regulation of rivers, improvement of harbors, construction of lighthouses to safeguard navigation, telegraphs, telephones, radio, metallurgical processes, steam power, machinery, automobiles, good artificial illumination, erection of great structures, manufacturing processes, and the like—play an important part in bringing people into more intimate personal contact by increasing community life or by enabling it to subsist, and by associating rural populations and community populations with each other. Machines provide the capstone in this development.

These engineering influences conjointly lifted the peasant

populations of Europe from serfdom to reasonable security and freedom—from sordid poverty and marginal living to conditions of relative security of livelihood and happiness—by making mutual contacts easy for the people, increasing mutually sympathetic relations, and assuring prompt transportation of goods when and where needed or desired. The added effects of great activity in scientific discovery and invention have brought for every group of our American people (except those in the richest groups) a higher level of living than has ever been attained in corresponding groups elsewhere. While we now enjoy these advantages compared with earlier centuries, the common people of nations which have failed to cultivate the opportunities of science and engineering still live a marginal life, suffering from the elements and from near-starvation. What is true in these contrasts between nations is also true in contrasts between industries in each nation.

Can we afford to slacken our efforts at the scientific discovery and invention which have produced so much for us and give promise of continued contributions of service? We want still better conditions for every one of us; and, particularly, betterment of the conditions of citizens who are in the groups of lowest welfare is wanted for them and by them. Hope for such results lies in improving the conditions of economical and plentiful production, and the historical precedents show that for such results scientific discovery and engineering invention are our reliance. It is right and natural for each individual to wish for better conditions and to the extent of his ability seek means for securing the fulfilment of the wish. The only useless human beings are those few who are on the bottom level of living conditions and have no wish to help themselves to go higher. They must be helped in some way; but how? That is a difficult problem to solve. Purely sentimental processes will not produce desirable results.

Engineering processes and engineering devices have been (and are being) abused by use in warfare, and much of the poverty now found in various nations is an aftermath of costly wars which sacrificed productive men and valuable property. This is not by choice of the leading minds of the engineering profession. The profession with few exceptions

stands on the conviction that, besides being inhuman, war is tragic, expensive, inefficient and unnecessary—tragic because of its track of human wastage, hardships and sorrows; expensive because of its wanton destruction of desirable wealth, which may be represented either by money expenditure or by property destruction; inefficient because irritating steps and high handed action, substituted for patience in negotiation for adjusting differences, seldom produce returns comparable with the costs. Hence the problem is to get all parties to use wisdom instead of folly, cease irritating acts, and put their differences to negotiation and impartial adjudication. The people of few nations (and those mostly among the prosperous small nations) seem to be ready for this. As world affairs display themselves, we therefore must be prepared in an organized way as a nation to stand against aggression from without, in case aggression should arise in spite of friendly negotiations planned to prevent forcible action.

If every man can ever be convinced that war will not ultimately result in greater wealth, security and stability for himself and his nation, the tendency toward organized war will be weakened. Especially will this be achieved if other sufficient routes are kept open on which men's love of risk and adventure can be fully expended, and the paths of selfish profit during war are closed. Individual quarrelling or collective war have always existed in the world of man; and they continue jointly with us today, as we know to our frequent regret. The results of scientific discovery and invention have given war a more comprehensive and frightful aspect; but they also have given greatly increased populations and largely magnified available wealth on the earth. Probably the proportional destructiveness of war is no greater today than it was millennia ago when entire populations were destroyed or converted into slavery as the consequence of wars.

It is not science or engineering which is at fault. The defects in human relations which arise in our day in local, national and international affairs are not due to faults in the fundamental ideal of a complete civilization, nor to a scientific engineering which has set up and continues to maintain the stage for a possible realization of that ideal. The fault lies in the lag of human intelligence with respect to its oppor-

tunities for bettering human conditions. Deficient business morals and spread of crime in a country like ours, and the insistence of sovereign governments of a right to do as they please without regard to generally accepted ethics or to solemnly made agreements, indicate a lack of confirmed principles of ethics held by the people and by government leaders. The human mind, in mass, is very slow to change in character; and stable improvements of mutual relations cannot be rushed to success by the vaguely applied efforts of professional reformers or dilettantes in social affairs. Such improvements may only be successfully produced slowly in association with sounder-growing mass-thought regarding living affairs in both material and ethical relationships.

Such processes are fraught with alternate states of success and discouragement. The influence of engineering is of primary importance for showing the ways to material betterment. The history of the nations which I have briefly sketched shows this. Following in the wake of material betterment and increased leisure which are secured by improved engineering practices, intellectual improvement also usually comes as an ultimate contribution to social welfare. I now put it to you: Is not the choice of occupation assumed for the hypothetical Benjamin Franklin of 1906 made in the best interest of humankind? Then also: Is not the interest of humanity favored by encouraging great continued activity in scientific discovery and engineering invention, while legitimate efforts are also maintained for the purpose of studying and ameliorating social defects?

INVITATION TO INDIVIDUAL THINKING.

I will leave you with this final suggestion: Clarence Day in his entertaining book called "This Simian World" says: "In a Simian civilization, great halls will be erected for lectures, and great throngs will actually pay to go inside at night to hear some self-satisfied talk-maker chatter for hours. Almost any subject will do for a lecture or talk; yet very few subjects will be counted important enough for the average man to do any *thinking* on them, off by himself."

With that merry paragraph in mind, I express the hope that this lecture may have the effect of stirring at least a few productive thoughts.

Air Conditioning Vital to Precision in Manufacture of Aircraft Engines.—(*Heating, Piping and Air Conditioning*, Vol. 12, No. 3.) The new plant of the Allison Div. of General Motors Corp. at Indianapolis, Ind., is the world's latest version of the air conditioned, artificially lighted, windowless industrial building. Its program for the manufacture of aircraft engines for the U. S. Army calls for a high degree of precision and careful handling of materials. Air conditioning is thus an essential in avoiding the effect of temperature changes on gages and precision machines, etching of highly polished metal surfaces because of perspiration, and for providing the comfortable conditions that are necessary because of the concentration and skill required of the workmen. The building and its equipment are unique in many respects; among its features are: (1) A combination of natural well water, "Freon" refrigerant, natural air and auxiliary oil burners in the air conditioning and heating scheme. (2) The first application of a new type of fluorescent lighting to a large scale industrial lighting problem duplicating mid-day north light 24 hours a day. (3) Because of the unique combination of factors in heat insulation, air conditioning and artificial lighting it will take relatively less power to create this new approach to ideal working conditions. (4) Unusual paralleling of a diesel power plant with local utility electric lines to provide an adequate, uninterrupted source of power with unusual economy. (5) Unusual architectural effect, due to windowless feature and use of glass brick panels. (6) An unusual tooling set up for precision work. Four new buildings on 53 acres of land comprise the plant; they include the manufacturing building with adjoining office building, a test building and a gate house. Welded construction of steel columns and trusses is used throughout.

R. H. O.

THE CINEMA INTEGRAPH.[1]

A MACHINE FOR EVALUATING A PARAMETRIC PRODUCT INTEGRAL.

BY

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WITH AN APPENDIX [1] BY

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INTRODUCTION.

In a previous issue of this JOURNAL,[2] a machine called "A Photoelectric Integraph" was described by T. S. Gray. This machine had been developed at the Massachusetts Institute of Technology [3] and was used to expedite the work of evaluating an integral of the form:

$$F(y) = \int_a^{b \text{ or } x} f(y \pm x)g(x)dx, \qquad (1)$$

where $g(x)$ and the kernel $f(y \pm x)$ are known functions, and y represents a variable parameter.

After this machine had been used for a few years to aid with the solution of various problems, it became apparent that it was potentially an effective aid in mathematical analysis but that its speed of operation and accuracy, while good, were not all that could be desired. Accordingly, construction was commenced on another machine embodying the same basic principle for performing the product integration

[1] The Cinema Integraph was developed in part by G. S. Brown in partial fullfilment of the requirements for a thesis. The appendix contains numerical examples taken from a thesis by W. R. Hedeman, Jr. on the more general subject "The Numerical Solution of Integral Equations on the Cinema Integraph." These theses were submitted to the Faculty of the Massachusetts Institute of Technology in partial fullfilment of the requirements for the Degrees of Doctor of Science.

[2] Gray, T. S., "A Photoelectric Integraph." *J. F. I.*, **212**, 77–102, 1931.

[3] The original basic suggestion for a machine of this type was made by Professor Norbert Wiener.

but having much greater scope as a mathematical tool. This machine is described in this paper and is called the Cinema Integraph.

The type of integral represented by Eq. (1) occurs frequently in electrical-network problems, in periodogram analysis, in the solution of problems leading to certain kinds of integral equations, and in several other problems. As represented by Eq. (1) the kernel $f(y \pm x)$ is a special form of the more general kernel $f(y, x)$. Integrals embodying this more general form of kernel occur frequently in the expansion of functions in terms of a series of orthogonal or bi-orthogonal functions, in transforms, in integral equations of potential-theory problems, in ship-stability studies, and also in numerous other important problems. Numerical solutions for illustrative problems involving each form of kernel are given in the appendix.

In most problems the evaluation of the integral is required for many values of the parameter y, but because of the large amount of tedious labor involved and the length of time required to complete the evaluation, some form of mechanical aid is usually essential if problems embodying this type of integral are to be solved frequently. The Cinema Integraph is designed to be such a mechanical aid that will be useful in many problems. In its present state of development it can handle the integral in an effective manner when either form of kernel is encountered. When the $f(x, y)$ kernel is encountered the machine evaluates the integral for discrete values of the parameter such as $y_1, y_2, y_3, \cdots y_n$, using successively the kernel functions

$$f(y_1, x), f(y_2, x), f(y_3, x) \cdots f(y_n, x),$$

with the values for y spaced as close together as is necessary. When the $f(y \pm x)$ kernel is encountered the machine gives the solution as a continuous function of the parameter y.

To perform the multiplication and integration process indicated by Eq. (1), the Cinema Integraph uses light from an incandescent source and an arrangement of masks. The light is measured by phototubes in a manner similar to that used in the earlier machine.[4] Motion-picture film is used for the

[4] Gray, T. S., loc. cit.

masks which transfer the functions $g(x)$ and $f(x)$ into the machine instead of the cardboard masks used in the earlier machine. From this use of motion-picture film comes the name "Cinema Integraph." An important feature of the procedure is that analytic expressions for the functions are not required; the only requirements being that it be possible to plot them on a two-dimensional graph with sufficient accuracy, that each plot be single valued, and that any discontinuities be finite in number.

The time required to evaluate a parametric product integral using the new machine is not very great. Briefly, it is necessary first, to prepare the function films from the graphs of the functions using an auxiliary apparatus provided, second, to locate these films in the machine, and third, to make a few adjustments to calibrate the machine for the particular problem and to interconnect certain mechanisms according to the form of solution desired. From then on the operation is almost entirely automatic and the complete solution is obtained within a few minutes and is usually accurate to within approximately one per cent. of full scale. The preparation of the films requires a time from one to several hours with the present equipment, the time being relatively short when the kernel is of the form $f(y \pm x)$, and almost proportional to the number of values of y for which a solution is desired when the kernel is of the form $f(y, x)$. Since certain kernel functions occur in many different problems the films for these functions, once prepared, can be used over and over again with consequent saving in time.

The time required to adjust and to calibrate the machine preliminary to running a solution is seldom in excess of 15 minutes. The entire machine procedure is relatively simple though a certain amount of care and skill are required when exposing and processing the photographic film.

To make the scope of the device greater than its predecessor, provision has been made so that the integral for one value of the parameter is given directly when either or both functions have positive and negative values. The solution is recorded automatically, usually in the form of a sequence of printed numbers when the kernel is $f(y, x)$, and as a curve when $f(y \pm x)$. When the kernel is $f(y, x)$ the successive

operations of recording the result, stepping the films to the next value of the parameter, recording, stepping, and so forth can be entirely automatic with the mechanisms provided. These recording-stepping cycles require a few seconds per cycle.

In order to indicate the matters that stimulated the development of a machine of this kind, as well as to facilitate a more concrete conception of the operations such a machine is called on to perform, a brief treatment of the method of procedure for the solution of a typical problem is given in the next section. Following this the details of the component mechanisms and the operation of the machine itself are described.

A TYPICAL PROBLEM INVOLVING A PARAMETRIC PRODUCT INTEGRAL.

The commonly encountered replacement problem [5, 6] is a typical example of many problems whose formulation leads to an equation of the type amenable to solution using the Cinema Integraph. This example is of interest, first because the method of attack is representative of that used for the formulation of an integral equation, and second because it illustrates the use and effectiveness of what may be called a "solving function" or "reciprocal" kernel.

A simple form of the replacement problem arises when a known number of objects of a given kind, such for example as lamp bulbs, is to be maintained in service. This known number may be a function of time having any finite value at time zero. Each object has a finite life and is replaced by a new one immediately upon failure. If the life expectancy or mortality-rate for these objects is known, the replacement rate necessary to maintain a given number in service can be calculated as a function of time. The replacement rate is not, of course, restricted to positive values only.

For a specific example, let electric incandescent lamps be the objects considered, and let

[5] Lotka, A. J.: "The Application of Mathematical Analysis to Self-Renewing Aggregates"; *Annals of Math. and Statistics*, March, 1939.

[6] The potential usefulness of the Cinema Integraph became apparent in the solution of replacement problems when Mr. P. A. Samuelson, Society of Fellows, Harvard University, asked Mr. W. R. Hedeman, Jr., and the authors what machines were known to them which were available for use on such problems.

$N(t)$ = a known function giving the total number of lamps required in service at time "t."

$S(t)$ = a known function giving the fraction of a unit number of "typical" lamps installed at time zero that survive an operating time "t."

$R(t)$ = the desired unknown function giving the number of new lamps put in service (additions and replacements) per unit time at time "t."

To obtain the desired unknown function $R(t)$, the procedure is to formulate an expression giving the number of lamps in service from all sources at a given time "t." Of those placed in service at time zero, namely $N(0)$, the number that will have survived to time "t" equals $N(0)S(t)$. Now during an interval dT at a time T between zero and "t," the new units placed in service as additions and replacements equals $R(T)dT$. At time "t" the age of these particular new units is $(t - T)$, and only the fraction $S(t - T)$ of them will have survived to this time. Thus, of the lamps added during the interval dT at time T, there are surviving at time "t," $R(T)S(t - T)dT$ lamps. If this expression is integrated over all possible values of T between zero and "t," and added to the number that were put in service at time zero and survive to time "t," the sum is the total lamps in service at time "t," that is

$$N(t) = N(0)S(t) + \int_0^t R(T)S(t - T)dT. \qquad (2)$$

Usually the total number of lamps $N(t)$ desired in service at any given time "t" is known, the life expectancy $S(t)$ of such lamps is determined by test of a representative sample group, and the unknown function is the replacement rate $R(t)$. Eq. (2) then becomes an integral equation of the first kind, Poisson type, for determining $R(t)$.

For certain forms of $N(t)$, and $S(t)$, the machine may be used to solve Eq. (2) as it stands. However, in this and in many other problems of this type, it is preferable to change Eq. (2) into a more convenient form. In this instance, differentiating Eq. (2) with respect to "t" and rearranging gives

$$N'(t) - N(0)S'(t) = S(0)R(t) + \int_0^t S'(t - T)R(T)dT. \quad (3)$$

Then, if $S(0) \equiv 1$,

$$N'(t) - N(0)S'(t) = R(t) + \int_0^t S'(t - T)R(T)dT. \quad (4)$$

Eq. (4) is known to have a solution of the form:[7, 8]

$$R(t) = N'(t) - N(0)S'(t) + \int_0^t \sigma(t - T)[N'(T) - N(0)S'(T)]dT, \quad (5)$$

where $\sigma(t)$ satisfies the equation

$$S'(t) + \sigma(t) = -\int_0^t S'(t - T)\sigma(T)dT. \quad (6)$$

The function $\sigma(t)$ depends only on the function $S'(t)$, that is, it is reciprocal to $S'(t)$ and is commonly called the *reciprocal kernel*, or *solving function*. The advantage of solving Eq. (6) in preference to Eq. (4) is that *once the reciprocal kernel corresponding to a given $S'(t)$ is determined by solving the integral equation (6) the function $R(t)$ for any given $N(t)$ can be calculated by explicit integration* using Eq. (5). This reciprocal kernel $\sigma(t)$ is independent of the number of lamps desired in service, being dependent only on the function $S'(t)$ of the particular type of lamp. Eq. (6) is an integral equation of the second kind, Poisson type, and its solution using the Cinema Integraph is a relatively simple procedure. When $\sigma(t)$ has once been determined for a given type of lamp and service, the replacement function $R(t)$ necessary to maintain in service a total number of these lamps $N(t)$ as any desired function of time is obtained merely by the computation of an explicit product integral. This integral is the kind which the Cinema Integraph handles directly and extremely easily.

To proceed with the problem two sets of data are required, the first set being that for $S'(t)$, or alternatively $S(t)$ from which $S'(t)$ can be found; and the second set being that for $N'(t)$ or alternatively the desired $N(t)$. The functions $S'(t)$

[7] Volterra, V.: "Leçons sur les Équations Intégrales," Gauthier-Villars, Paris, 1913.

[8] Bôcher, M.: "An Introduction to the Study of Integral Equations," Cambridge, University Press, 1926.

and $N'(t)$ are ultimately to be plotted in the form of curves, and can be given in either tabular or graphical form. The data for plotting these curves are the only ones required before the machine can be used.

It is readily recognized that the solution of a problem of the kind represented by Eq. (2), either by solving Eq. (2), or Eqs. (6) and (5), is an exceedingly laborious task if some machine aid is not available. Using the Cinema Integraph, however, the time required to solve and to check the solution of a problem for which the functions $S'(t)$ and $N'(t)$ are given is estimated, as a result of performing an actual solution, at not more than 10 to 14 hours. This time includes determining the reciprocal kernel $\sigma(t)$ and compares favorably with the time requirement of many days using generally available methods. Once this $\sigma(t)$ is obtained for a given $S'(t)$ the time required to obtain solutions for each different $N'(t)$ is probably not in excess of an hour or so.

A numerical solution for this type of problem with given $S(t)$ and $N(t)$ functions, as well as the solution for another illustrative problem involving the $f(x, y)$ kernel is given in the appendix by W. R. Hedeman, Jr.

PRINCIPAL MACHINE MECHANISMS.

In describing the manner whereby the machine performs the multiplication and integration process, and in discussing the important aspects of the machine operation, it is convenient to refer to certain component mechanisms in terms only of their function or characteristic action. These principal mechanisms, and their functions are briefly:

(1) A line source of radiation for providing the light whereby is performed the multiplication and integration process.
(2) A mechanism for locating in the machine the films which represent the functions and for moving these films through the machine with either intermittent stepping, or continuous motion.
(3) An optical system comprising apertures, mirrors, lenses and light-integrating spheres for directing into a photoelectric receiver the light which represents the value of the integral.

(4) An auxiliary light source and a mechanism for measuring the light which represents the value of the integral.
(5) A photoelectric receiver and vacuum-tube amplifier for controlling the automatic operation of the light-measuring mechanism.
(6) A servo motor controlled by the vacuum-tube amplifier for driving the automatic light-measuring mechanism.
(7) A device coupled to the light-measuring mechanism for recording the solution of the problem, either as a curve or as a table of numbers.
(8) A relay-control apparatus for the automatic sequencing of various operations performed during a solution.
(9) A device, called a film exposer, for preparing the films on which the functions are represented.

Figure 1, a photograph of the Cinema Integraph, shows certain of these mechanisms. The line-filament lamp, optical system, and film-drive mechanism are located in the body of the machine, and the servo motor for driving the light-measuring and recording mechanism is located in the base of the machine. The vacuum-tube amplifier and electrical relay-control mechanisms are mounted on the rack shown at the left of the main machine.

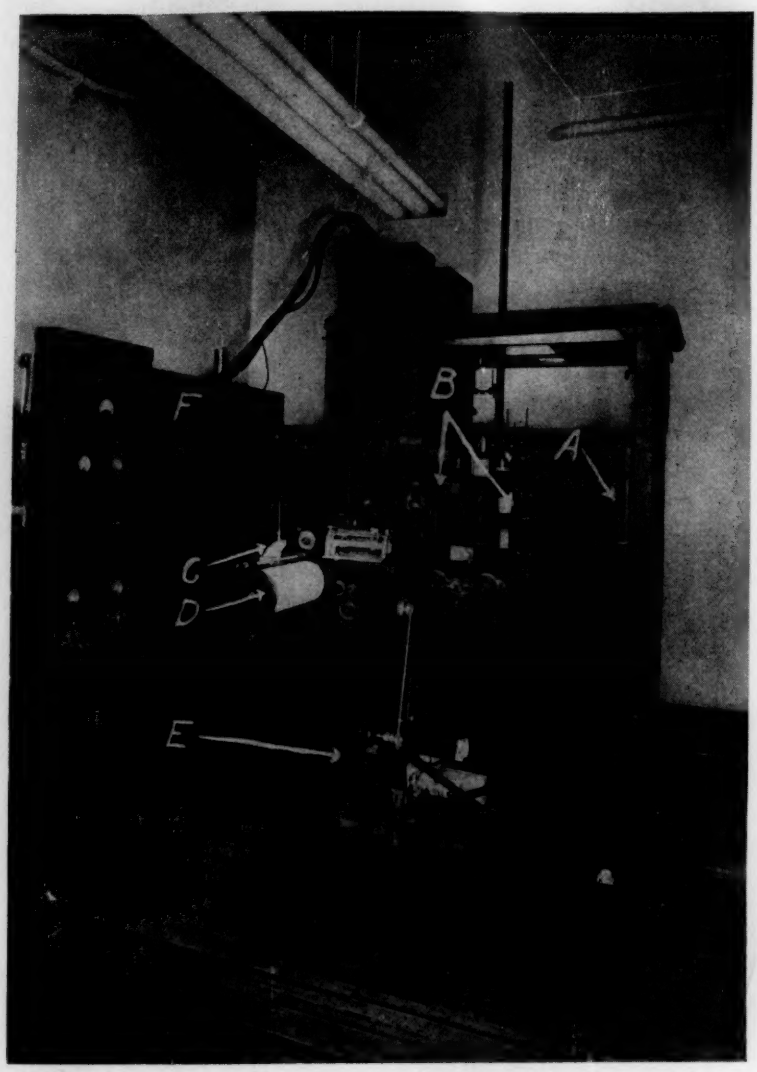
THE MULTIPLICATION AND INTEGRATION PROCESS.

(1) Basic Principle.

The principle by which the machine multiplies two functions and integrates their product is basically the same as that used in the Photoelectric Integraph. This principle is demonstrated by reference to Fig. 2 which represents a simplified schematic drawing of the portion of the machine wherein the multiplication and integration processes are performed. In this figure *SS'* represents a vertical line source of radiation [9] having substantially uniform radiation

[9] The envelope of the line source is an evacuated glass tube about 24 inches long and 2 inches inside diameter, ground and polished inside and outside. Mounted within the tube is a straight tungsten wire 0.010 inch diameter which is electrically heated to 2100° to 2400° K. For details of filament-temperature control see "A 5-ampere electronic D-C regulator" by J. N. Coombs and P. T. Nims, Electronics, p. 40, Jan. 1940.

FIG. 1.



Photograph of Cinema Integraph showing principal mechanisms. *A*. Line source of light. *B*. Driving sprockets. *C*. Recording counter. *D*. Recording drum. *E*. Servo-mechanism. *F*. Relay control panel.

per unit length along its length. In the explanation that follows the line source is considered to radiate light with directional characteristics given by Lambert's Law. The light from the source falls on the rectangular aperture A_1, which is located at the point e' along the optical axis e, e', e''. This aperture is in the form of a rectangle, whose top and bottom edges are horizontal, and whose vertical edges lie in a vertical plane perpendicular to the optical axis e, e', e''. A second rectangular aperture A_2, is located on this optical axis so that the distance e to e' equals the distance e' to e''. Each aperture lies in the surface of a right circular cylinder whose axis coincides with the line source. The points e' and

FIG. 2.

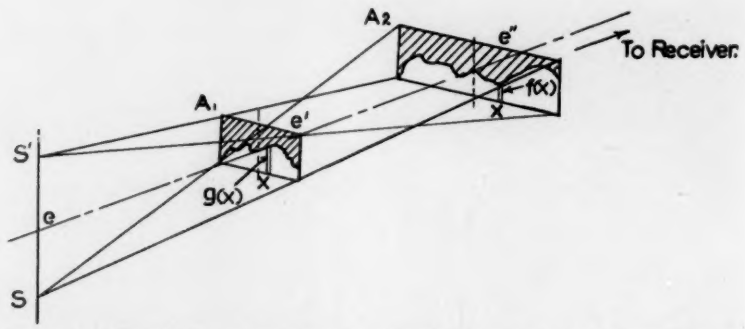


Diagram for demonstrating multiplication and integration processes. SS' Line filament lamp; A_1, A_2 Function apertures.

e'' are in the center of the apertures A_1 and A_2 respectively. Any given edge of the aperture A_2 and the corresponding edge of the aperture A_1 lie in the same plane.

Each function $g(x)$ and $f(x)$ of Eq. (1) is transferred into the machine on a strip of photographic film prepared with transparent and opaque regions so that an ordinate of the transparent region represents the corresponding ordinate of the curve of the function, and the opaque region represents area that lies above the curve. Thus, the boundary between the transparent and opaque region represents the plot of the function. Suppose for simplicity that both functions have values of only one algebraic sign. The film representing the function $g(x)$ is inserted in the aperture A_1. Similarly,

the film representing the function $f(x)$ is inserted in the aperture A_2. The axis of x lies along the horizontal edge for each aperture, and the films are located along this axis so that any plane that contains the line source intercepts the two films at equal values of x.

The *multiplication process* is visualized by considering a single, narrow, vertical element of a transparent region on the $g(x)$ film of width dx at the abscissa x. Suppose that this element consists of a vertical chain of small rectangular holes, with each hole of width dx, and height dg, and with the total height of the chain equal to $g(x)$. The light transmitted by *each* hole forms a vertical line image of the line-source of light on the surface of the aperture A_2, at the same value x of abscissa, after the style of a pin-hole camera. The intensity of illumination of this image is proportional to dg, and the ratio of aperture height to aperture separation is made sufficiently small so that the intensity of illumination on any part of the line image appearing at the aperture A_2, due to any small hole on the aperture A_1, is essentially uniform everywhere along the ordinate.

The total illumination incident on the illuminated vertical element at x on A_2 is proportional to the combined height of the pin holes that constitute the vertical element of the $g(x)$ function, because the illuminations of the images superpose linearly. That is, the total illumination at this vertical strip on A_2 is proportional to the total height of the transparent ordinate on A_1, or proportional to $g(x)$. At the abscissa x on the aperture A_2 the height of the transparent strip is $f(x)$. Therefore, the elementary component of light flux passing through the vertical strip of width dx at the abscissa x on aperture A_2, is proportional to the product of the illumination at this abscissa and the height of the strip; that is, to $g(x)f(x)dx$.

The *integration process* is visualized if this reasoning is extended to include all the transparent strips that together compose the functions $g(x)$ and $f(x)$. The total light flux passing through the whole aperture A_2 is proportional to the sum of all the elementary products $g(x)f(x)dx$, that is to

$$\int_a^b g(x)f(x)dx, \qquad \text{for } a \leq x \leq b. \tag{7}$$

The limits of integration $b - a$ correspond to the range of x equal to one aperture width.

(2) **Provision for Handling Functions with Positive and Negative Values.**

As shown in the preceding section, the integral of the product of two functions is translated by the machine into a quantity of light flux. In general the functions $g(x)$ and $f(x)$ may both have both positive and negative values. The product of these functions can be represented symbolically as:

$$g_{\pm}(x)f_{\pm}(x) = \begin{cases} g_{+}(x)f_{+}(x) = \text{positive component of the integral} \\ g_{-}(x)f_{-}(x) = \text{positive component of the integral} \\ g_{+}(x)f_{-}(x) = \text{negative component of the integral} \\ g_{-}(x)f_{+}(x) = \text{negative component of the integral,} \end{cases}$$

where, for example,

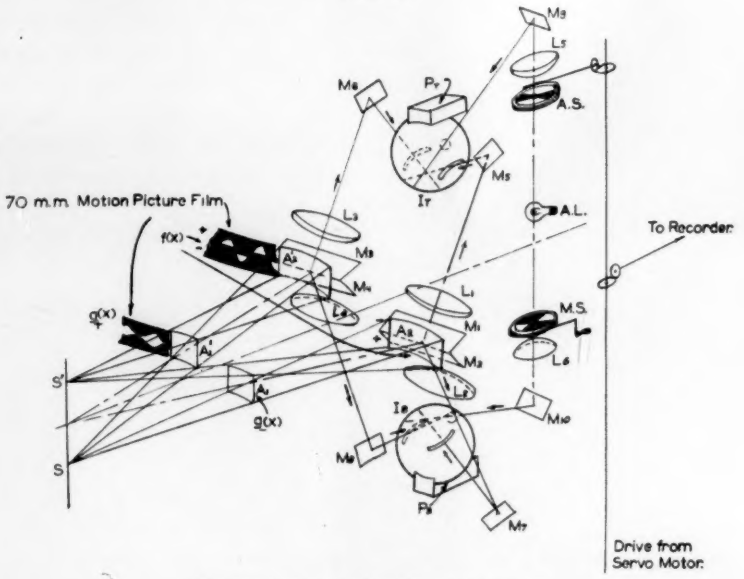
$g_{+}(x)$ signifies only the positive values of the function $g_{\pm}(x)$,
$g_{-}(x)$ signifies only the negative values of the function $g_{\pm}(x)$.

The integral is given by the machine in terms of two components of light flux representing the positive contributions to the integral, and two components representing the negative contributions to the integral. The value represented by the algebraic sum of these components is measured photoelectrically, and indicated automatically.

When both functions involve both positive and negative values the film inserted in the aperture A_1 is prepared so that it is transparent for values of $g_{\pm}(x)$ of only one algebraic sign, say the positive sign $g_{+}(x)$. The film for the function $f_{\pm}(x)$ is prepared to represent both the positive and the negative values of the function with the axis of abscissas running horizontally along the center of the film. It is inserted in the aperture A_2 with the region above the axis of abscissas representing positive values of the function. The light flux transmitted through the aperture A_2 above the axis of abscissas then represents a positive contribution to the integral, since it represents the product $g_{+}(x)f_{+}(x)$. The light flux transmitted below the axis of abscissas represents a negative contribution to the integral, since it represents the product $g_{+}(x)f_{-}(x)$. These films make available only two of the required four components of light flux.

To provide the other two components of light flux a duplicate set of apertures A_1' and A_2' are provided, as shown in Fig. 3. A film is prepared so that it is transparent for only the negative ordinates of $g_{\pm}(x)$, namely $g_{-}(x)$, and it is inserted into the aperture A_1'. A film identical in form to the film of $f_{\pm}(x)$ used in the aperture A_2 is inserted into the aperture A_2', but here it is turned upside down so that the transparent regions representing the negative portions of

FIG. 3.



Schematic diagram of complete optical system.

$f_{\pm}(x)$ are above the axis of abscissas. The light flux transmitted through the aperture A_2' above the axis of abscissas now represents the other positive contribution to the integral, since it represents the product $g_{-}(x)f_{-}(x)$. The light flux transmitted below this axis represents the other negative component, since it represents the product $g_{-}(x)f_{+}(x)$.

Figure 3 shows a schematic diagram of the optical system of the machine. By means of the mirrors M_1 and M_3 located behind and above the abscissa axes of the apertures A_2 and A_2' respectively, the two positive components of light flux are

directed upward onto the mirrors M_5 and M_6 which reflect this flux into an optical integrating sphere [10] I_T. A phototube P_T receives light from the sphere I_T. Similarly, by means of the mirrors M_2 and M_4 located behind and below the abscissa axis of apertures A_2 and A_2' respectively, the two negative components of light flux are directed downward onto the mirrors M_7 and M_8, which in turn reflect this flux into an integrating sphere I_B. The phototube P_B receives light from the sphere I_B. Thus, the four components of light flux are first segregated and then the two positive components are totalled in the sphere I_T and the two negative components totalled in the sphere I_B. Hereinafter the components of light flux representing the total positive contributions to the integral are referred to for convenience as *positive light flux* or *positive light*. Similarly the components of light flux representing the total negative contributions to the integral are referred to as *negative light flux* or *negative light*.

(3) Measurement of Light Flux.

The measurement of the algebraic sum of the components of light flux is accomplished by means of an auxiliary light system used in conjunction with a null-indicating photoelectric bridge. The phototubes P_T and P_B that see the light are of the vacuum type, and are connected in series and form the balancing arms of the bridge. The bridge is in the null or balance condition when equal light flux enters each phototube. To bring about this balance condition, a fixed light flux equal to the maximum positive or negative light flux that can be transmitted by the films, is admitted into the integrating sphere I_B and thence to the phototube P_B. This light flux, called the *bias light*, is supplied by the lamp AL, Fig. 3 It is maintained constant Also, a balancing quantity of light flux, called the *balance light*, is supplied by the lamp AL and directed into the integrating sphere I_T and thence to the phototube P_T This light flux is variable in

[10] Integrating spheres are used so that the non-uniform surface-response characteristics of the phototubes play a negligible role in the machine operation. The inside wall of the sphere is sprayed with a magnesium oxide and water mixture and afterwards smoked with burning magnesium. The procedure gives a rugged, efficient, highly diffuse reflecting surface.

magnitude from zero up to a value equal to twice the amount of the bias-light flux by means of a calibrated light shutter. This shutter is herein called the *balance-light shutter* or *balance shutter*.

The balance or equilibrium condition of the phototube bridge occurs when the positive light plus the balance light equals the negative light plus the fixed bias light. Hence, when the positive and the negative light fluxes transmitted by the apertures A_2 and A_2' are equal, the balance light must equal the bias light for the bridge to be in the balance or equilibrium condition. The position of the balance shutter which accomplishes this therefore corresponds to the zero value of the integral. If the positive light is greater than the negative light the balance shutter must transmit less light to preserve the balance condition. To do this it must be moved toward the closed position, and its displacement toward the closed position from the position of zero integral is then proportional to the net positive value of the integral. Similarly, if the positive light is less than the negative light the balance shutter must be opened by a certain amount and transmit more light to restore the balance condition. Its displacement toward the open position from the position of zero integral is then proportional to the net negative value of the integral. A recording mechanism is driven by a shaft coupled to the mechanism that operates the balance shutter.

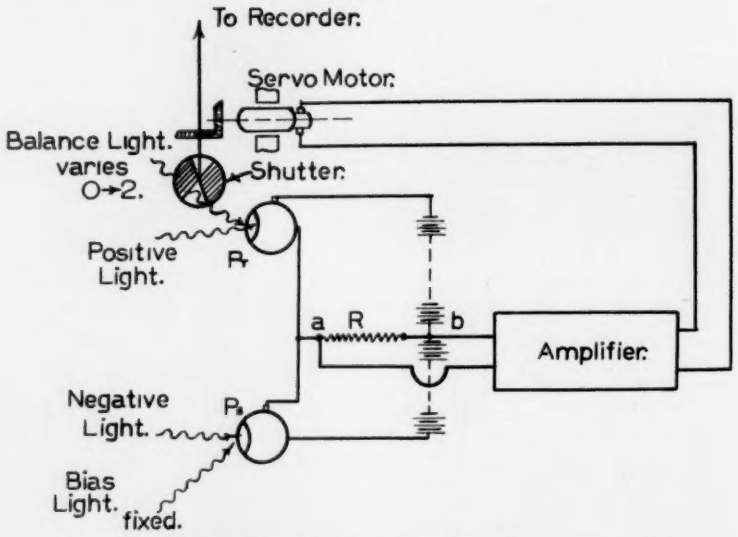
The adjustments to the position of the balance shutter, and hence the operation of the recording mechanism, do not require the attention of the person operating the machine. They are performed automatically by a small high-speed servo motor.[11] The automatic operation is achieved by connecting the measuring arm of the phototube bridge to the input terminals of a d-c. vacuum-tube amplifier, which in turn supplies direct current to the armature of the servo motor. The schematic arrangement of phototubes in the bridge, amplifier, servo motor and balance shutter is shown in Fig. 4. Actually the servo motor operates the first stage of a me-

[11] Hazen, H. L.: "Design and Test of a High-Performance Servo-Mechanism," *J. F. I.*, **218**, No. 5, pp. 543–580, Nov. 1934.

chanical torque amplifier,[12] not shown in Fig. 4, which supplies the relatively large torque necessary to operate the drive shaft of the balance-light shutter and the recording mechanism.

When the light fluxes entering the phototubes depart from the equilibrium condition, a voltage signal is given to the amplifier which operates the servo motor in a direction that changes the amount of light transmitted by the balance shutter so as to restore the condition of light equilibrium. By adjustment of the amplifier gain this photoelectric servo-

FIG. 4.



Schematic diagram of automatic light-measuring device.

mechanism can be operated,[11] over a wide range of light levels employed, with less than critical damping, critical damping, or more than critical damping, whichever is desired.

OPERATING PROCEDURE AND FORM OF SOLUTION.

In order to obtain the solution of a problem on the machine the following operations are performed.

1. Placing the functions $f(x)$ and $g(x)$ together with suitable calibration functions on their respective films.

[12] Nieman, *American Machinist*, **66**, pp. 895–897, May 26, 1927.

2. Adjusting the machine so as to give a desirable range for the balance shutter displacement.
3. Determining the calibration constant of the light measuring and recording mechanism.
4. Setting the film-drive and recording unit controls to function automatically with the correct sequence of operations for the problem in hand.

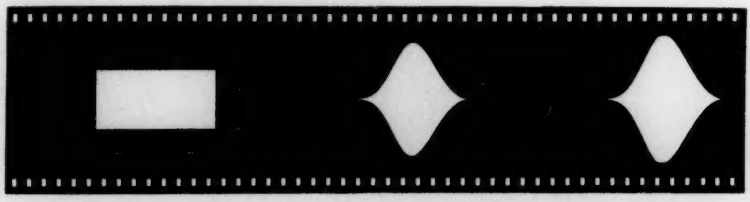
Each of these operations will be explained in more detail under its heading.

1. *Placing the Functions on the Films.*—This operation is performed semi-automatically, using a film-exposing mechanism described in the next section, from graphs of the functions plotted on an 18″ × 24″ plotting table. The choice of scales for the original graphs, as well as their reproductions on film, is based on experience and is influenced by the shape of the particular functions involved, an effort being made to avoid excessively great slopes because of the machine errors inherently associated with them.

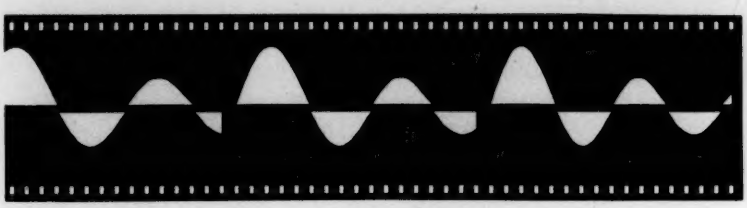
The films are 70 mm. motion-picture stock with perforations along each edge, and are accurately positioned in the machine apertures by means of toothed drive sprockets. See Fig. 5.

FIG. 5.

a.



b.



Typical sections of function films. These are made from contact prints of an actual film.
(*a*) Film for aperture A_1 with a rectangular calibration section. One sign of function only.
(*b*) Film for aperture A_2 with both signs of function.

On both the $f(x)$ and the $g(x)$ films one or more calibration sections are placed. In these calibration sections the function is usually a constant for convenience. Thus, in the $f(x)$-film the calibration sections carry the function

$$f(x) = C_f = \text{constant.}$$

Similarly, in the $g(x)$-film, the calibration sections carry the function

$$g(x) = C_g = \text{constant.}$$

The values of C_f and C_g are usually chosen so that

$$C \equiv \int_a^b C_f C_g dx \tag{8}$$

is approximately equal to the largest value expected during the solution of

$$\int f(x)g(x)dx,$$

as determined from a rough estimate made prior to the preparation of the films. The values of a and b are any convenient known limits, usually chosen so that $b - a$ occupies the entire aperture length.

Calibration sections are usually located at each end of each film and when a particular piece is longer than about three feet additional calibration sections are distributed along the film. In this way, the tendency for any variation in transparency along the film can be measured, and if a small variation is detected an appropriate small correction can be made to the solution.

2. *Adjusting the Balance Shutter Range.*—With calibration sections in both apertures, the filament voltages of the main and the auxiliary lamps are adjusted so that the balance shutter is at nearly its maximum displacement. Since the calibration functions have been chosen to provide a calibration integral approximately equal to the maximum expected value of the integral to be evaluated, this adjustment assures that the results will not fall off scale, and at the same time assures that the precision of the shutter settings will be a maximum.

In some problems the transparent areas on the films

represent such a small fraction of the area of the apertures that the main optical system transmits so little light that an undesirably low voltage on the auxiliary lamp appears necessary. The necessity to operate at this low voltage can be avoided, however, by the insertion of a suitable aperture stop in the auxiliary light system. This so reduces the transmission of the auxiliary system that the auxiliary lamp can be operated at a reasonable brightness with the balance shutter operating over its full range.

If the function carried on the film in aperture A_2 has both positive and negative values one further adjustment must be made. The sensitivity of the light-measuring bridge for the positive and negative contributions of light from aperture A_2 must be made equal. This is usually done by adjustment of the voltages on the two photocells, nearly identical cells having been selected initially.

3. *Determining the Calibration Constant.*—When the adjustment just described has been made, the reading of the balance shutter scale is noted with the calibration sections still in the apertures. Denote this reading by R_c. If the scale reading obtained with any $f(x)$ and $g(x)$ functions in the apertures is say R, it follows that

$$\frac{R}{R_c} = \frac{1}{C}\int_m^n f(x)g(x)dx, \tag{9}$$

that is,

$$\int_m^n f(x)g(x)dx = \frac{CR}{R_c} = KR, \tag{10}$$

in which

$$K \equiv \frac{C}{R_c},$$

and is the calibration constant of the machine for the particular adjustment. The values of m and n are the desired limits of integration.

A precise variable-ratio drive is now inserted in the recorder mechanism in such a manner that K can be given any desired value within certain limits. The result R can be read on a pair of dials, printed automatically on a paper tape, plotted automatically on a sheet of graph paper, or obtained simultaneously by any combination of the three methods.

4. *Setting the Automatic Controls for the Problem.*—Here a distinction is made between the procedure used for the kernel $f(x, y)$, and that for the kernel $f(y \pm x)$, because the type of film drive and recording is different for the two forms of kernel. Controls suitable to each form are provided. The function of these controls can best be understood after outlining certain details concerning the films.

When the integral has the $f(x, y)$ kernel, that is, when

$$F(y) = \int f(x, y)g(x)dx, \tag{11}$$

the film for $f(x, y)$ includes as many separate sections or *frames* as there are values of the parameter y for which $F(y)$ is desired. These frames occupy adjoining precisely located positions on the film as in the usual moving-picture film. In addition calibration sections are included at convenient intervals.

It is assumed that as a result of the preceding operations both the f and g films have been properly inserted in the machine with their calibration sections centered or "framed" in their respective apertures, and that the value of R_e is ready to be recorded. This can be done by pushing a button. Pushing another control switch steps both films precisely one frame, bringing the $g(x)$ function and the first frame of the $f(x, y)$ function into the apertures. Thereafter the sequence is as follows: (1) the value of $F(y)$ for the first value of y is recorded, (2) the $f(x, y)$ film is then stepped one frame to the second value of y, (3) the result of (2) is recorded, (4) the film is stepped ahead to the third value of y, and so on until the entire solution is recorded. This sequence is performed quite automatically, and from this discrete-step movement of the kernel film the $f(x, y)$ kernel is frequently referred to by machine operators as the *stepping* kernel.

In more detail, consider that the initial frames $f(x, y)$ and $g(x)$ are in position, that the automatic controls have been properly set, and that the "execute" button has been pressed. With the film-drive sprockets held hard against positioning stops, the servo-motor is driven until the balance shutter has balanced the light on the two photocells. When this balance is reached within a prescribed small tolerance the servo-motor

armature current has decreased to such a value that a relay is operated which short-circuits the servo-motor armature, and thus prevents its moving appreciably. The same relay actuates the printer which prints R for the particular $F(y)$. Following the printing operation another relay causes the $f(x, y)$ film to be moved automatically until a contact on the film-positioning stop indicates that the film has completed its one-frame motion or step. This contact opens the short-circuit on the servo-motor armature, thus allowing the motor to drive the balance shutter to a new balance position corresponding to the new value of $F(y)$. Thus a cycle is completed, and is repeated automatically with a period of a few seconds, until terminated either automatically after a predetermined number of cycles or until stopped manually. The result is a printed table of $R = \frac{F(y)}{K}$ for the values of y for which the $f(x, y)$ film was made.

When the integral has the $f(y \pm x)$ kernel, the procedure is modified somewhat from that given above. For example, let

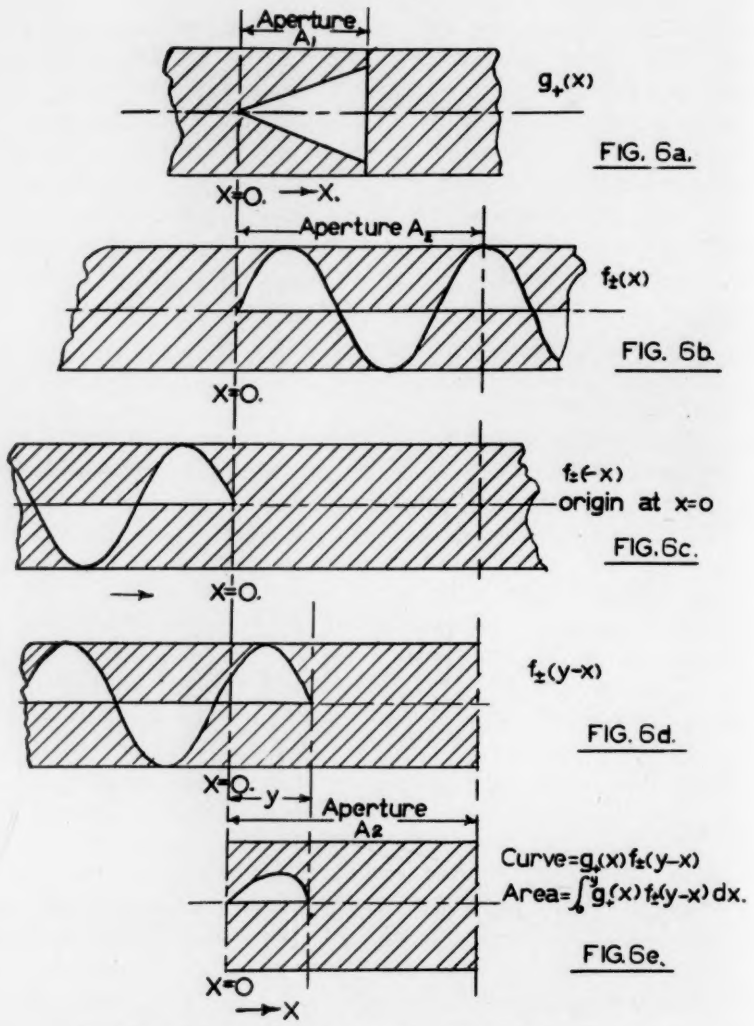
$$F(y) = \int f(y - x)g(x)dx. \qquad (12)$$

The $f(y - x)$ film is here represented not on separate frames for different values of y but as one continuous function of y, perhaps extending the length of several frames. This film is in fact merely the $f(x)$ film, Fig. 6*b*, turned end for end about the origin, giving $f(-x)$ as shown in Fig. 6*c*. When its origin is displaced by an amount y, Fig. 6*d*, the portion in the aperture represents $f(y - x)$.

It will be apparent that this kernel film should not be stepped through the machine frame by frame but shifted continuously, whence the value of $F(y)$ can be determined as a continuous function of y. Thus, referring again to Fig. 6, as the $f_{\pm}(-x)$ film is moved into the aperture in the direction of increasing x, the displacement of its origin from the point $x = 0$ established by the $g(x)$ film corresponds to the value of y. The movement of this film is made gradual and continuous, and the servo-motor keeps the balance shutter continuously in the position of light equilibrium. The displacement of the shutter is then proportional to the value of the integral

$$\int_0^y f_\pm(y - x)g_+(x)dx, \tag{13}$$

FIG. 6.



Graphical illustration of method for handling kernel of the form $f(y - x)$.

as a continuous function of the parameter y An adjustable-speed motor drives the mechanism that moves the film into

the aperture. Because of this continuous shift of the kernel film, the kernel $f(y \pm x)$ is frequently referred to by machine operators as the *shifting* kernel.

To record the solution a curve-drawing stylus on an output platen is connected to the machine so that its abscissa is proportional to y and its ordinate is proportional to the displacement of the balance shutter. This stylus traces the curve of the continuous solution $F(y)$ as a function of y. In problems where a printed record of the solution is preferred, the movement of the $f(-x)$ film can be stopped at the desired intervals of y and the result printed on the tape by the recording counter. The above procedure is readily altered to handle the problems where the kernel is of the form $f(y + x)$.

Although the procedures have been described principally in terms of one set of apertures, namely A_1 and A_2, the use of both sets introduces no new ideas since the film-drive sprockets for the A_2 and A_2' apertures are geared together and therefore move synchronously as do those for the A_1 and A_1' apertures. Separate films are of course required for all four apertures as described earlier in the paper.

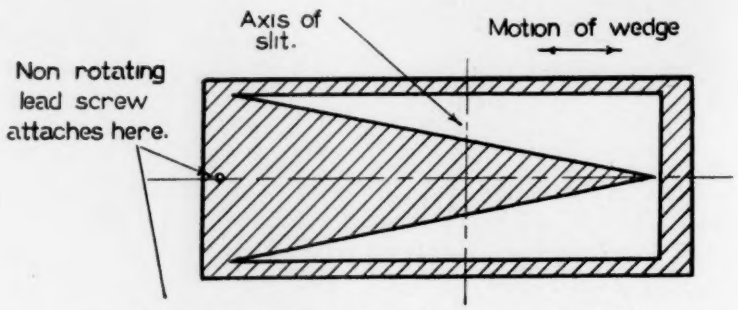
FILM EXPOSER.

The curves of the functions are transferred onto photographic film on a device called a film exposer. This device serves to admit light to the surface of the film so that, after processing, the region above the curve of the function becomes opaque and the region below the curve becomes transparent. The boundary between the opaque and transparent regions then represents the curve. The mechanism comprises essentially (1), a table or platen on which are located the plots of the functions, (2), an index pointer placed above the platen and driven by abscissa and ordinate lead screws to follow the curve and (3), a film-exposing chamber. The film-exposing chamber comprises (4), a means for moving unexposed film beneath an illuminated slit as the index pointer on the platen is driven parallel to the abscissa axis of the platen, and (5), a means to cover a length of slit proportional to the ordinate position of the index pointer on the platen. The ordinate position of the index pointer is constrained by a hand crank to

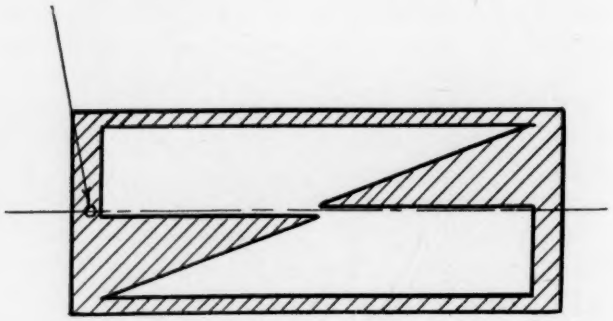
follow the curve as the pointer is moved parallel to the axis of abscissas, hence the width of film rendered unexposed is proportional, except for a slight correction mentioned below, to the ordinate of the function. The film-drive shaft and the abscissa-drive shaft of the index pointer are coupled together and are motor driven.

The means for covering a varying length of the slit consists of a flat wedge-shaped mask which is moved perpendicular to the direction of the slit and parallel to the movement of the film by a screw. This screw is driven by a gear train coupled to the ordinate drive of the index pointer. The wedge-shaped mask is referred to as a wedge, and for the usual problems two forms of mask are used. The one for preparing the films for the apertures A_1 and A_1' is shown in Fig. 7*a*, and that for preparing the films for the apertures A_2 and A_2' is shown in Fig. 7*b*.

FIG. 7*a* and *b*.



7*a*. Wedge provided to prepare films for front apertures A_1 and A_1'.



7*b*. Wedge provided to prepare films for rear apertures A_2 and A_2'.

It is important to notice that the mask of Fig. 7*a* is in the zero position when the apex of its wedge is at the edge of the exposing slit, and that it makes the film transparent for equal distances on each side of the center of the strip of film. This procedure is not only permissible from the mathematical standpoint, but highly desirable from a practical standpoint because it decreases appreciably an error, resulting from the compactness of the design, in the multiplication of the functions as performed in the machine.

The mask of Fig. 7*b* used to prepare the films for the apertures A_2 and A_2' is in fact a "wedge pair." It is in its zero position when the adjacent apexes of the wedge pair are beneath the exposing slit. Reference to Fig. 5*b* shows that the films for use in these apertures have an exposed region running down their center. This separates the axis of abscissas for the positive values of the function from the axis of abscissas for the negative values by a 3/32 inch opaque zone. This is done, first because a finite width of mirror surface on each side of the common edge of the mirror-pairs M_1, M_2 and M_3, M_4 of Fig. 3 is imperfect owing to slight chipping of the glass at the edge during handling, and second, because the spacing between the common edge of each mirror-pair and the plane of its corresponding aperture would otherwise have to be so small—to separate the light accurately into its components—as to be impractical.

A matter that justifies mention here concerns a slight curvature applied to the edge of each wedge. Because of the ratio of aperture height to aperture spacing required for a reasonably compact design with good illumination efficiency, the intensity of illumination on the rear aperture is less at the top and bottom edges than in the center. If the curves on the film are undistorted reproductions of the original graphs, an error is inherent in the multiplication process performed by the machine. To reduce this error the edges of the wedges are curved so as to add to the transparent ordinate on the film an increment that amounts to approximately one per cent. at maximum ordinate, and which varies approximately as the fourth power of the ordinate between zero and maximum. An interesting commentary on the overall precision of the Cinema Integraph is given by the fact that since this error

had been expected to fall below the probable precision the wedges were originally made without correction, but that a small consistent systematic error disappeared from results after the theoretical correction to the wedges was made.

To permit a wide choice of scales for the plots of the functions, adjustable-ratio gear trains are provided to couple the ordinate and abscissa lead screws on the platen with the wedge-drive screw and the film-movement sprocket respectively in the exposer. A relay-control apparatus actuated by cams mounted on the shaft driving the film-movement sprocket is provided to facilitate accurate starting and stopping of the abscissa drive shaft.

(To be continued in August issue.)

AN ASTRONOMICAL HYPOTHESIS TO EXPLAIN PERMIAN GLACIATION.

BY

RONALD L. IVES.

ABSTRACT.

Speculation concerning possible causes of Permian circumequatorial glaciation has resulted in several interesting hypotheses, among which are those postulating various continental drifts, and several based on assumed changes in atmospheric circulation occurring as a result of relatively minor changes in the size, relative location, or interconnection of the continental masses.

An astronomical hypothesis, attributing the known climatic peculiarities of the early Permian to the effects of a ring of small satellites about the earth in approximately the plane of the equator, is here presented. This, for convenience, will be referred to as the Ring Hypothesis.

INTRODUCTION.

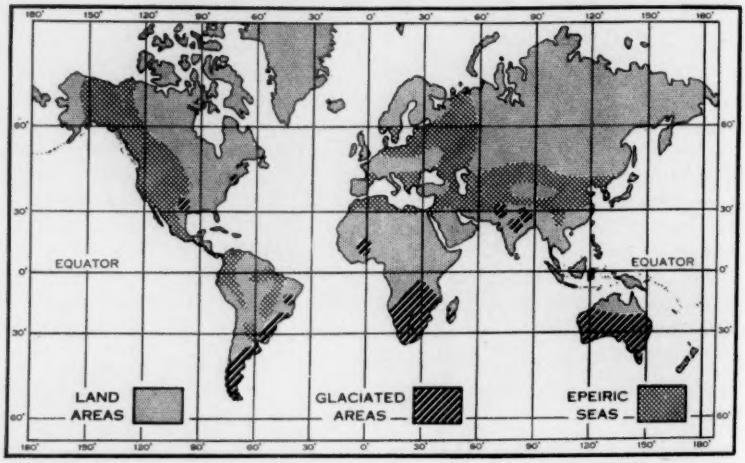
Of the four major ice ages [1] known to have occurred during earth history, three can be attributed, on the basis of evidence now available, to a general cooling of the earth's surface, and explained, in part at least, by several promising theories, such as Humphreys' "inverse hothouse" effect,[2] or Simpson's theory of the climatic effects of variations in solar radiation.[3] Evidence of several world-wide lowerings of temperature during the Pleistocene is so complete and detailed as to be practically incontrovertible: that supplied by Huronian and late pre-Cambrian deposits is less complete.

The remaining, and most severe, ice age, that of the Permian, cannot readily be explained on the basis of general terrestrial cooling alone. Maximum glaciation, during this interval, took place in areas in, or very close to, the present tropics.

Location of ice, land, and water, during the Permian ice age, in areas now land, is shown in Fig. 1. This map, generalized from those of Brooks, Coleman, Schuchert, and Willis,[4] shows approximate conditions. Many of the details are disputable, as a result of fragmentary evidence and inadequate field work, and no two workers are in exact agreement on all

features. Most, however, are agreed that Permian glaciation took place in about the areas shown, and that the major glaciations were approximately simultaneous. Some geolo-

FIG. 1.



The world, on Mercator's projection, showing condition of present lands during the early Permian, as generalized from maps of Brooks, Coleman, Schuchert and Willis.

gists believe that the ice ages were late Pennsylvanian, rather than early Permian, a difference of opinion largely caused by the extreme difficulty of exactly dating land deposits.

Possibility that the Permian glaciation covered a much larger area than is indicated by present evidence is suggested by Coleman,[5] who commented (1926) that in South Africa "It looks very much as if only the southern half of the glaciated area had been discovered." Since that time, evidence for a northward extension of the African glaciated area has been found. Further investigations in South America might have a similar result.

THE SPECIAL PROBLEM OF THE PERMIAN ICE AGE.

Any area, to be the source of a glacier of any type, must have an annual average snowfall in excess of average annual melting and sublimation. If an ice field is to increase in size, snowfall on it must increase, or attrition decrease. This,

according to generally-accepted theories, is most simply brought about by a lowering of temperature, which, all other factors remaining the same, causes a greater part of the annual precipitation to fall as snow, and inhibits melting. When, as happened in the Pleistocene, ice creeps equatorward from circumpolar regions, and downward from the highlands, the glacial advance can be attributed to a general chilling of the earth's surface.

In the early Permian, ice apparently crept toward the present poles from regions now circumequatorial. This seemingly anomalous ice motion certainly cannot be explained as a result of general cooling *alone*. Some change other than, or in addition to, a general cooling of the earth's surface must have taken place during, or just before, early Permian to produce widespread glaciation in areas now temperate or tropical.

Numerous hypotheses have been advanced to account for Permian glaciation. Of these, two general types are still under consideration—those in which the glaciated lands are assumed to have been near the poles during Permian time, and to have subsequently moved equatorward (continental drift); and those postulating that snow-producing cold and moisture were carried to the glaciated regions by air and ocean currents, largely confined to the southern hemisphere by land barriers of various sorts (land-bridge hypotheses).

CONTINENTAL DRIFT HYPOTHESES.

To account for Permian glaciation, it has been suggested that the glaciated lands, while occupied by ice, were located in polar regions, and have since drifted elsewhere.

Various similar, but by no means identical, hypotheses of continental drift have been presented by Wegener,[6] DuToit,[7] Taylor,[8] and others.

Wegener, whose hypothesis of continental drift is perhaps the best known and most bitterly disputed, contends that a primal continent, Pangæa, was formed from a thin world-wide granitic crust some time before the Silurian. After various latitudinal and other peregrinations, Pangæa reached the Antarctic zone during early Permian, and was intensely glaciated. At the beginning of the Jurassic, Pangæa split up into fragments, which today, after various wanderings and some

further fragmentation, form the continents. In the gaps which formed between the separating fragments of Pangæa the Atlantic, Arctic, and western Indian Oceans grew at the expense of the Pacific. According to this hypothesis, continental drifting, which Wegener attributes to tidal forces, should still be going on. Measurements of the supposed drift of Greenland, a project which cost Prof. Wegener his life, have not yet produced conclusive results.

DuToit's hypothesis, largely based on that of Wegener, calls for two primal continents, Laurasia in the north and Gondwanaland in the south, separated by a persistent intercontinental geosyncline. Although the assumed motions of the continents and their fragments are somewhat similar to those suggested by Wegener, DuToit attributes them to an entirely different set of forces—cyclonic disturbances of the convectional circulation in the sub-crust—which he considers roughly analogous to similarly-named atmospheric disturbances.

Taylor, who first introduced the concept of continental drift, assumes that the drift-producing forces are tidal, and that drifting began in the Cretaceous, when, according to his hypothesis, the moon was captured and became a terrestrial satellite with a very eccentric orbit.

Daly [9] postulates a downhill sliding of the continental masses as the cause of drift, a concept similar to older views expressed by Suess [10] and Argand.[11]

A possible factor in continental drifting is described by Joly,[12] who suggests that intermittent retention and escape of radioactively-generated heat deep in the earth leads to variations in the fluidity of deep-lying rock layers, and would lead to both continental drifting and deformation of continental masses.

Numerous other continental drift hypotheses have been presented from time to time. Some of these are more or less reasonable modifications of current ideas; others, in their attempts to explain all evidence, postulate continental reshufflings similar to the gyrations of ice floes in a tide-rip.

A vast amount of evidence bearing on continental drift has been collected, and a not inconsiderable part is favorable to one or another of the hypotheses. Objections to the various

hypotheses have been numerous. Lack of exact correspondence of the margins of continents once adjacent, according to the hypotheses, and now separated, has often been pointed out. Lithologic or biologic dissimilarities of areas supposedly once in contact are numerous, and if actual, rather than apparent because of inadequate field evidence, constitute a serious objection to the hypotheses.

Geophysical arguments are almost unanimously opposed to the concept of continental drift, the tidal forces postulated by Taylor and Wegener as motivators of the drift being entirely inadequate. Jeffreys [13] suggests that tidal forces are about ten billion times too small to produce the motions postulated by Wegener in the time he allows, and that if tidal forces adequate to produce this motion were applied, they would stop the earth's rotation in about a year!

Joly's hypothetical intermittent fusion of the substrata as a result of radioactive heating seems contrary to some of the laws of thermodynamics, according to Jeffreys.[14] DuToit's postulated cyclonic disturbances of the convectional circulation of fluid substrata are interesting, but until more data concerning the present and past conditions in the substrata are gathered, neither this hypothesis nor Joly's can be fully evaluated. There is no assurance that the substrata are fluid, or have been so within the last half billion or more years.

Despite its present condition, which may be likened to that of a jigsaw puzzle of which some of the pieces are lost, and others don't fit together very well, the concept of continental drift is still a "live issue"; and, regardless of its ultimate fate, a valuable stimulus to geological and geophysical research.

LAND-BRIDGE HYPOTHESES.

So many temporary continental interconnections have been proposed in the past to account for distribution of life that land bridges have been called the last refuge of the paleobiologist. Although recent studies of variations in sealevel have removed the apparent necessity for many of these bridges, a few are still retained by those paleobiologists who do not accept one or another of the continental drift hypotheses.

Recently, land bridges, in conjunction with a general low-

ering of temperature, have been proposed to explain the rather peculiar distribution of ice during the Permian glacial period.

Within the last few years, Brooks,[15] Schuchert,[16] and Willis,[17] working respectively from paleoclimatic, paleobiological, and geophysical data, have developed surprisingly similar, but by no means identical, hypotheses to explain the climates of the Permian.

These hypotheses require land bridges, during the early Permian, which largely isolated the waters of the South Atlantic and Indian Oceans from the other seas, creating a landlocked body of water, which, as a result of a slight general lowering of temperatures, was refrigerated by Antarctic ice. Arctic waters, at this time, were confined to the far north by land bridges linking Europe, Greenland, and North America, and blocking the Bering Strait. According to these hypotheses, waters of the combined South Atlantic and Indian Oceans could only come into contact with those of the other seas by means of a wide channel between Australia and Antarctica, while Arctic waters reached those of the Tethys Strait through the Volga Sea.

These hypotheses, explaining in general, although not in detail, the climates of the early Permian, have much in their favor. Under them, the Permian glaciations of South America, Africa, and Australia are quite logically attributed to chilling and precipitation at the contact of cold air from the region of the landlocked southern sea with warmer air from the lands.

The explanation of the Indian glaciers, as a result of chilling of land air by cold ocean currents that travelled across the equator from Antarctica without being appreciably warmed, is hardly credible. Neither Schuchert nor Willis satisfactorily explains the cause of the North American glaciation that produced the Squantum Tillite, near Boston.[18] Brooks postulates a narrow seaway extending from the Arctic Ocean to latitude 53 degrees to produce refrigeration in New England, an assumption not wholly justified by field evidence.

Many of the other arguments opposed to these hypotheses are based on doubtful, nebulous, or incorrect evidence, although the erratics of the Caney shales of Oklahoma, if their

supposed glacial origin and Permo-Pennsylvanian age is verified, constitute a definite objection to both the land-bridge and continental drift hypotheses.[19, 20, 21]

Recent studies indicate that land bridges, called "Isthmian Links" by Willis, are not geophysically impossible, as was once thought, and that they may be formed from the sea floor in the same manner that mountain ranges are upraised from the continents. As the subsea portions of an isthmian link are protected from subaerial erosion, traces of all former links should still exist on the sea floor, and should be detectable by soundings. Such seems to be true with the postulated Brazil-Guinea Link.

Jeffreys [22] suggests that changes in the volume of crustal materials, such as might occur if 20 km. of tachylyte crystallized to eclogite, would be adequate to founder any land bridge yet postulated, without creating isostatic unbalance.

Further researches, such as the subsea geophysical work of Meinesz, the sea-floor core-punchings by Piggot, and studies of dredged-up microfossils by Lohman and others, seem to be in order before the general concept of isthmian links, successor to the older idea of land bridges, can be completely evaluated.

Quite obviously, the isthmian links postulated by Willis cannot have been coexistent with the Gondwanaland described by Wegener, even though quite similar, and sometimes identical, evidence is used in support of both hypotheses.

THE RING HYPOTHESIS.

Introductory Statement.

Another approach to the problem of the Permian Ice Ages is possible. If, instead of a general refrigeration accompanied by either continental rearrangements or rerouting of air and water circulations, a reduction of the insolation received by lands in low latitudes only, is postulated, most of the now-known climatic changes of the early Permian can be explained.

Causes of Equatorial Cooling.

None of the postulated causes of the Pleistocene ice age is competent to produce cooling largely confined to regions near the equator. To bring about this rather restricted lowering of temperature, some sort of shading medium, either in the earth's atmosphere or in space, seems necessary.

The continued existence of a shading medium in the earth's atmosphere over equatorial regions only is mechanically impossible. A circumequatorial belt of volcanic dust high in the atmosphere would, if the particles were of the proper sizes, produce a lowering of temperature under it, but researches by Humphreys [23] show that volcanic dust, wherever produced, spreads through the stratosphere in a few months, and would ultimately produce a general, rather than a restricted, temporary reduction in the insolation received at the surface. A permanent cloud belt over the tropics would likewise cool the lands under it, but after a slight initial lowering of temperature, sufficient, perhaps, to change the climate of the polar regions, but not that of the tropics, from non-glacial to glacial, the effect of a cloud bank becomes "hothouse," rather than "inverse hothouse."

Numerous types of shading media in space are possible, but no general cloud in space could produce the sharply-defined cooling necessary to account for the Permian ice age. The shading medium, whatever its configuration, must cast a relatively "sharp" shadow on earth. The existence of such a medium in space, beyond the earth's gravitational influence, for a period long enough to permit the accumulation of ice sheets of continental dimensions, and its subsequent vanishment without leaving any recognizable evidence of its existence other than Permian glaciation, seems improbable for mechanical reasons. Close to earth, however, and occupying an orbit around it, a cloud of matter capable of casting a shadow of the requisite sharpness, and of enduring through the millennia necessary for the accumulation of great ice sheets, could exist, could vanish without leaving direct geological or astronomical traces, and would have a configuration quite similar to the Saturnian ring system. Upon the supposed existence of such a ring during the early Permian, and its effect on terrestrial climates, the Ring Hypothesis is based.

Origin of the Ring.

Any body captured by a larger body either falls upon its captor or assumes an orbit about it. The capture of a satellite by a planet probably requires energy changes such as would be produced by the presence of a resisting medium

about the captor, in addition to the action of gravitative forces. The eccentricity and circumference of the new satellite's orbit may be altered by frictional, gravitational, or tidal influences; its plane by frictional and gravitational or tidal forces.

As the orbit of a satellite decreases for any reason, the disrupting effect of tidal forces on the satellite increases, until, at a certain point, called the Roche Limit,[24] disrupting forces exceed cohesive forces, the satellite is eventually torn apart, and the fragments form a ring about the primary, generically related to the Saturnian ring system, believed by many astronomers to be the wreckage of a satellite.

When a ring of fragments occupies an orbit, collisions between the fragments tend to reduce the energy of the system (unless the fragments are perfectly elastic) and hence to bring it into increasingly closer proximity to its primary.

Further disruption of a satellite of relatively small size is brought about by the alternate heating and cooling of its surface as it travels about its orbit. This process is similar to the exfoliation of terrestrial rocks as a result of the relatively minor diurnal temperature changes, and should be greatly intensified outside the earth's atmosphere. When the satellite is so close to its primary that an eclipse of some duration occurs at each revolution, this type of disruption should become of great importance. If, however, the satellite is large enough to have an appreciable gravity of its own, and is not in frequent collision with other bodies having quite similar orbits, a protective layer of fragmented material will form over its surface, be held there by gravity, and will effectively shield the inner parts from further thermal fragmentation. Collisions, it should be obvious, would cause this protective layer to be rearranged or thrown off, exposing new surfaces to the full effects of temperature change. Where collisions are possible, this "skinning" of small satellites may be a more important factor in their final destruction than the fracturing resulting directly from collisions.

In obedience to these rules, a ring composed of myriad small satellites could conceivably be formed about the earth by at least three methods—the capture of a cloud of cosmic dust, the disruption of a captured satellite, or the destruction of an "original" satellite.

Capture of a Cloud of Cosmic Dust.—Should earth encounter a cloud of dust in space, perhaps a swarm of hitherto uncaptured "chips from creation," the head of a very dense comet, or the debris resulting from the collision of two planetoids, some of the particles would fall to earth, others would be too far from earth to be influenced by it and would continue their wanderings through space, and still others, under certain conditions, could become terrestrial satellites. The myriad small captive particles would tend to combine into a very small number of very much larger bodies under the action of gravitational and other forces [25] unless tidal action intervened. Hence, unless the dust particles came within the Roche Limit before assuming orbits about the earth, a new satellite, or group of them, instead of the desired semi-opaque ring, would be the direct result of the capture by earth of a cloud of cosmic dust.

This mode of origin of the hypothetical terrestrial ring seems least likely of the three suggested, but cannot be entirely ruled out. The chief objections to it are that "chips from creation" would be unlikely to remain as a cloud through nine-tenths of geologic time and all of "azoic" time [26] unless some unknown dispersing force counteracted the attractive force of gravity. A general cloud, also, unless quite small, and hence probably too small to contain adequate opaque material, would be attracted by and to the sun, as well as toward the earth, temporarily altering the solar constant, with resultant general changes in terrestrial climate. Such a cloud has been suggested by Shapley [27] as a possible cause of temporary general climatic changes. Comets, so far as is known, contain too little material to form a ring dense enough to produce measurable climatic changes on earth. Debris resulting from a collision of asteroids or other small bodies in the solar system could supply adequate opaque material, but the probability of its capture is small, and such a collision hundreds of millions of years after the formation of the solar system seems even less probable, although a large meteorite could conceivably knock a small asteroid out of its orbit and give the resulting swarm of fragments an earthward motion.

Capture of a Satellite.—During its travels about the sun, earth could capture a satellite from space, provided such a

body existed, came within the earth's attraction, and met certain other conditions, concerning which astronomers are not in exact agreement. Capture of its two diminutive moons, Phobos and Deimos, by Mars, has been suggested by Asaph Hall, their discoverer.[28]

The captive, for reasons to be discussed later, could be of small dimensions, would need to assume an orbit within the moon's, and probably one so close to earth that its period of revolution did not exceed a terrestrial day. This acquired satellite might be a stray from the junkyard of space composing the belt of asteroids, a giant meteor, or some small mass "shed" from the periphery of some unknown planetary system.

Capture of a satellite by a planet quite early in the history of the solar system seems entirely reasonable. After the lapse of two billion or so years between "creation" and the early Permian, however, such an occurrence seems less credible, although by no means impossible. A number of small bodies now following rather nonstandard orbits through the system, such as the asteroids Hermes and Eros, may some day be captured by planets.

Capture of the moon during Cretaceous time is postulated by Taylor [29] to explain his theory of continental drift.

A Second "Original" Satellite.—Two moons, one very small and close to earth, the other our present satellite, may have revolved about earth from the time of origin of the earth-moon system until the early Permian, at which time the minor companion, which, for convenience, will be called *Ephemeron*, drawn inward and disrupted by tidal forces, became the ring which caused the Permian circumequatorial glaciations. Destruction of an "original" small satellite seems the most likely cause for the ring, although neither a captured satellite nor an acquired cloud of cosmic debris may be entirely eliminated as a possible source of material.

No one theory of the origin of either the solar or the earth-moon system is as yet generally accepted by geologists, yet under none of the theories considered by them is the concept of a second terrestrial satellite impossible. Satellites, according to the Chamberlin-Moulton planetesimal hypothesis,[30] formed about minor condensation nuclei which were originally,

or soon became, gravitationally related to the larger nuclei. These larger nuclei, after growth by accretion, became the planets. Some astronomers question the origin of the moon in this manner, suggesting that it is a captured minor planet. Others believe that satellites are created by tidal disruption of their primaries. Darwin [31] and Jeffreys [32] both advance tidal hypotheses to explain lunar origin, Jeffreys attributing the fission of the earth to tidally accentuated oscillations of the molten sphere. The validity of the data upon which these two hypotheses are based is now open to question, largely as a result of recent research. Hypotheses suggesting a special creation for the moon do not in any way interfere with the concept of a second terrestrial satellite, and, in fact, suggest additional possible origins for it.

The presence of the moon, largest known satellite in relation to its primary, greatly complicates the existence of a small satellite between it and the earth, and suggests a decidedly eccentric orbit for Ephemeron, a condition hastening its destruction.

Orbit of Ephemeron.—Calculation of the various orbits of Ephemeron from the time of its acquisition by earth until that of its disruption by tidal forces cannot be done with any assurance, for neither the time nor mode of origin of this hypothetical satellite is definite, and its assumed density may be varied within wide limits without causing any fundamental change in the Ring Hypothesis. Fortunately, calculation of exact orbits is not necessary for the development of this hypothesis.

For Ephemeron to be tidally disrupted in early Permian time it must then have come within the Roche Limit. Accordingly, its position prior to the Permian must have been such that it would be drawn toward this limit, and hence earthward. Calculations by Darwin [33] and Jeffreys [34] show that any satellite having a period of revolution shorter than that of the rotation of its primary will be drawn toward the primary by tidal forces. This suggests that Ephemeron, during its life as a terrestrial satellite, had a period never exceeding earth's period of rotation, and was, if pre-Permian conditions of the earth-moon system were approximately the same as present conditions, never more than about 26,000

miles from the earth's center. Jeffreys' equations further show that, all other things being equal, the inward motion of a small satellite will be very much slower than that of a larger body occupying the same initial orbit. Hence, a small satellite, captured just prior to the early Permian, would occupy an orbit almost exactly at the Roche Limit, while a larger body, accompanying the earth from the time of formation of the system, could have an initial orbit with a radius somewhere between the Roche Limit and 26,000 miles from the earth's center.

The plane of Ephemeron's orbit, during the later stages of its existence at least, is assumed to have coincided approximately with that of the earth's equator.

Lunar attraction, as previously mentioned, would tend to make Ephemeron's orbit highly eccentric, causing the satellite to pass within the Roche Limit sooner than if the orbit had the same mean radius and a lesser eccentricity. Perturbations would undoubtedly increase the stresses tending to disrupt Ephemeron.

During the latter part of its existence as a satellite, Ephemeron would, if its motion were "direct," rise in the west and set in the east, cause an eclipse (total or annular, depending on the satellite's size and distance) of the sun somewhere on earth each day, and spend part of each night in eclipse.

Destruction of Ephemeron.—After an unknown interval of slow shrinking, the orbit of Ephemeron would come within the Roche Limit, and the satellite would be torn apart by tidal stresses. This destruction need not have been either sudden or spectacular, but could have taken place rather gradually, the surface "skin" of thermally exfoliated material, held in place only by the feeble gravity of the small body, being first removed. Later, as the disrupting forces overcame the mechanical strength of core of the satellite, this would be torn apart, and, as time went on, the fragments of the core would in turn be pulled to pieces. These larger fragments could have existed as independent, but doomed, satellites for a considerable interval of time after the initial breakup—perhaps for several million years.

Ephemeron, as will be explained later, could not have been very large. As a result of its small dimensions, the materials

composing the satellite could not have been appreciably compressed by gravity, and no great change in the density of the component materials would accompany its disruption. After the initial breakup of such a satellite, according to Jeans,[35] ". . . the orbits of the inner fragments will all be within the radius of limiting stability (Roche Limit), so that for each fragment the same process must repeat itself indefinitely, a limit only being reached when the fragments are so small that their chemical cohesion is able to defy the disruptive effects of gravitation and rotation. The outer fragments, on the other hand, will describe orbits which all lie outside the radius of limiting stability, so that they will not suffer further disintegration *at first* (italics mine). But the perihelia of these orbits are already very close to the sphere of limiting stability, and if the agencies which drove the original satellite inside this sphere are still operative, it may be expected that before long the new satellites also will be driven in and broken up in turn."

In the case of the hypothetical satellite, Ephemeron, the agencies (tidal friction, coupled with a period of revolution shorter than that of earth's rotation) referred to by Jeans continue to operate after the initial breakup, until all fragments of the demolished satellite have come within the Roche Limit.

In addition to the tidal forces tending to disrupt the satellite will be the effects of temperature change, already mentioned, and the stresses introduced by librations, which will somewhat resemble those produced in a cork which is rolled under foot. These and numerous other factors may lead to a considerable difference between the actual "radius of disruption" and Roche's theoretical value.

The direct product of the destruction of Ephemeron is a ring of fragments about earth, in, or very close to, the plane of the equator. This ring should closely resemble the present ring system about Saturn. It is interesting to note that whatever the source of the material; whether a swarm of fragments that took up orbits about earth within the Roche Limit, or from the destruction of a satellite or group of satellites, however acquired; a ring is produced. This ring eventually disintegrated, the component particles slowly falling to earth.

Composition and Size of Ephemeron.—The exact composition of the hypothetical second moon, Ephemeron, probably can never be determined. Inferences concerning its possible approximate composition are based on negative evidence only. Astronomical considerations show that its density could not have exceeded 60 times that of water, and was not less than 0.298 times that of water. The Roche Limit for a body having a density of 60 or greater falls within the earth's outer atmosphere, an environment unfavorable to both the formation and continued existence of a ring of minute satellites. Any body having a density of less than 0.298 would have a Roche Limit beyond the radius within which tidal forces give an inward motion, and if, for any reason, the body were shattered, the fragments would be moved outward, away from the earth, by these forces, and not only would no ring be formed, but the body would tend to be reconstructed!

The density range of from 0.298 to 60 may be immediately narrowed down, for no element denser than 22.5 (osmium) is known on earth. Hence the limits of Ephemeron's *possible* density are 0.298 and 22.5, a range of about 78 to 1.

Should Ephemeron have been composed of materials either not original to, or rare on, the earth's surface, this material should be present, or present in abnormal quantities, in Permian sediments (and to a lesser degree in sediments derived from them). No rare element has been found in Permian beds only, nor do rocks of that age differ chemically from those stratigraphically above and below them, so far as present evidence goes.[36]

Studies of meteorites show that the range of their densities is from that of granite (2.64) to that of iron (7.86), stony bodies probably being most numerous. As meteorites are believed to be made of the same materials as the planets and satellites, the *probable* density of Ephemeron was between 2.64 and 7.86, a range of approximately 3 to 1.

That Ephemeron could not have been a large satellite is again shown by negative evidence. A large satellite, on approaching the earth, would set up great tidal stresses in the crust, causing an extreme amount of seismic activity. No evidence of widespread abnormal terrestrial unrest during or just before the early Permian has yet been reported. A very

large satellite, on disintegrating, would eventually deposit an appreciable layer of debris on the earth's surface. This layer, even though it were slowly laid down, would decelerate erosion of the rocks on place; its components, being unconsolidated, would be easily removed, and would accumulate in basins, where evidence of it would surely have been found by geologists, had it existed.

From these considerations it may be concluded with some confidence that Ephemeron could not have been large, that it was not composed of materials uncommon on earth, and that its density was most probably between 2.64 and 7.86. It should be noted that the possible composition of the cloud of cosmic dust, suggested as an alternative source of the ring's material, is limited by the same type of evidence as is the composition of Ephemeron.

Size and Location of Ring.

It is assumed that the plane of the ring coincided with, or was very close to, the plane of the earth's equator. Slight variations from this plane might be expected as a result of various frictional and gravitational influences.

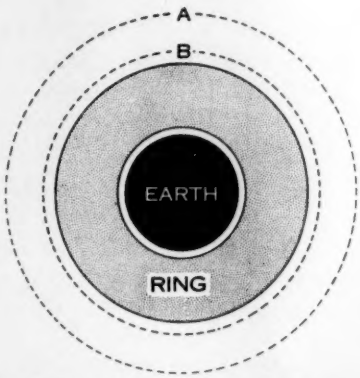
Regardless of the ring's radius, the orbits of its components, as a result of mutual attractions, would tend to be co-planar, limiting the thickness of the ring to a small fraction of its width, a condition prevailing in the now-existent Saturnian ring system.

The inner boundary of the ring would be somewhat indefinite, being determined by the "outer edge" of the earth's atmosphere. The atmosphere actually has no sharp outer boundary, but at a distance of about 500 miles above the earth's surface its pressure is less than that in the most "perfect" vacuum yet produced artificially, and little resistance should be offered to the motion of even a small, rapidly-moving satellite.[37]

The outer boundary of the ring would fall just outside of the "radius of disruption," which, in the absence of better data, is assumed to have Roche's theoretical value, and hence is determined by the density of the ring's component particles. Assuming that the average density of the particles was between 2.64 and 7.86, the outer boundary of the ring would be

between 12,250+ miles (density 2.64) and 8,660+ miles (density 7.86) from the earth's center. These limits are shown diagrammatically in Fig. 2.

FIG. 2.



Earth and its hypothetical Permian ring drawn to scale. The blank space between the inner margin of the ring here shown and the earth is occupied by the atmosphere. The outer limit of the ring shown is the Roche limit for an iron satellite. Dotted circle A is the Roche limit for a granite satellite, while B is that for a satellite having the same density as earth.

The size of the particles forming the ring depends upon the mechanical strength of the component materials. It seems probable that no particle much larger than a baseball could long remain unshattered under the combined effects of tidal stresses, repeated collisions, and the great variations in temperature resulting from frequent eclipses.[38] If the particles were small enough to remain suspended in the air for any appreciable time after striking it, they would be carried through the atmosphere by its convectional circulation, and a general Permian chilling would be produced. As there is very little good evidence for this, it may be assumed with some confidence that the majority of particles composing the ring could not have been very much smaller than grains of sand at the time they fell into the stratosphere. On similar grounds, it may be concluded that the particles could not have been composed of materials that disintegrated on being heated by atmospheric friction.

During the life of the ring, there should have been a continual rain of particles into the atmosphere. This, during nights in the early Permian, must have produced extremely

spectacular meteoric effects, resembling a rain of fire in the upper atmosphere over the equatorial regions.

The volume of material contained in the ring was undoubtedly small. Saturn's ring is believed to have a mass not over 0.00001 that of its primary,[39] and a ring about earth of dimensions comparable to Saturn's would have a far greater shading effect on the lands under it than is necessary to account for the climates of the Permian. A ring composed of materials as dense as the earth, and having 0.00001 of the terrestrial mass, would result from the tidal disintegration of a satellite having a radius of less than 85 miles. Should the particles be of the proper sizes, and have orbits in a suitable configuration, the disintegration of a satellite as small as Phobos[40] might supply enough material to produce the climatic changes in question.

Climatic Effect of the Ring.

When a ring exists about a planet in approximately the plane of its equator, while the planet's axis of rotation is not perpendicular to the plane of the ecliptic, the ring casts a shadow on the surface of its primary. The total area of the ring's shadow, and the part of the planet shaded, will vary seasonally, the shadow being densest when its area is smallest. Thus, a dense shadow over a small equatorial band will exist at the equinoxes, and a more extensive, but less dense, shadow will be characteristic of the solstices.

Such a ring is postulated to explain the climatic peculiarities of the early Permian. The origin of the ring, with several alternatives, has already been discussed. That the earth's axis, during the early Permian, was not perpendicular to the plane of the ecliptic is shown by the existence of seasons at that time, a phenomenon not readily explainable on any other basis than obliquity, and amply evidenced by the presence of varves in early Permian glacial clays and by annual rings in plants of that age.[41] The actual obliquity of the earth's axis to the plane of the ecliptic may not, and need not, have been exactly the same as at present. This hypothesis only requires that an appreciable obliquity exist, a requisite met by both sedimentary and paleobotanical evidence.

Climate is the resultant of average atmospheric conditions,

and is primarily a function of the radiation received by the earth and of latitude. Redistribution of heat received from the sun, the stabilizing effect of large bodies of water, and the varied effects of local topography are important secondary climatic influences.

The earth today is divided into three major climatic zones—tropical, temperate, and polar. In the tropics, incoming radiation is considerably in excess of outgoing terrestrial radiation, and temperatures here are reduced to values noninhibitory to life only by outward and upward migration of heated air. Air warmed in, and migrating poleward from, the tropics is called tropical air. In the polar zones, outgoing radiation is greatly in excess of incoming. Here, life is only possible on the equatorward edges of the zones, and in favored locations where temperatures are elevated by incoming warm air and water. Air cooled in, and migrating equatorward from, the polar zones is called polar air. Between the tropical and polar zones, incoming and outgoing radiation are approximately equal, and here the tropical and polar air masses come into contact, producing the storms so characteristic of the temperate regions.[42]

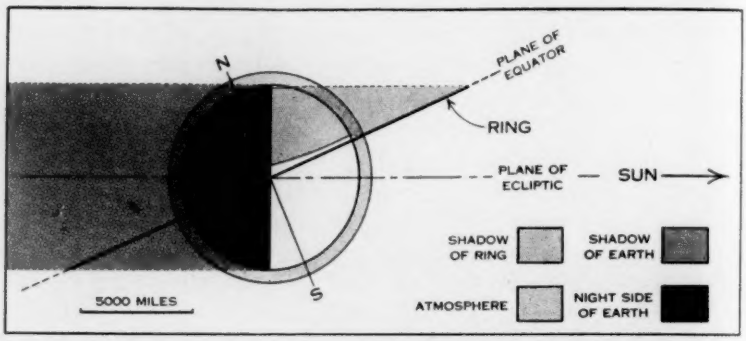
During the early Permian, three types of air masses—tropical, polar, and subannular—are postulated, instead of the two now existent. In the shadow of the hypothetical ring, source area of the subannular air mass, incoming radiation would be reduced by the ring's opacity, while outgoing radiation, until it passed beyond the earth's atmosphere, at least, would not be interfered with. Hence, subannular air would be cold.

Conditions on earth during the existence of this postulated ring are shown in Fig. 3. This, a scale elevation, with the earth's atmosphere and the shadow of the ring in space shown in cross section, illustrates the relative sizes of the various elements at the time of the winter solstice in the northern hemisphere, obliquity being assumed to have been the same as at present.

The presence of the hypothetical ring introduces many complicating factors into even the most simplified consideration of climates. Because of the gap, caused by the earth's atmosphere, between the earth's surface and the inner margin

of the ring, the equator, and a region half as wide as the ring's thickness on each side of it, would have two periods of minimum insolation, one at each equinox. At all other times, one hemisphere would have summer, with approximately

FIG. 3.



Scale elevation of the earth, at the time of the winter solstice in the northern hemisphere, during the existence of a hypothetical ring having an outer radius of 9,750 miles.

normal insolation, while the other would have winter, intensified by cooling as a result of the ring's shadow. The width of the zone of extra cooling—the source of the subannular air mass—would continually vary, and at each equinox would shift from one side of the equator to the other.

In the summer hemisphere, a line of storminess would exist, as now, at the contact of the tropical and polar air masses. A second line of storms would be present at, or very near to, the equator, where the warmest air mass—the tropical—came into contact with one very much colder—the subannular. If the subannular air were cold enough, a belt of continual snowfall would occupy the contact zone. Summers, under the conditions outlined, would not be exactly as present summers, for during the life of the ring the tropical air mass would tend to be smaller, and a considerable part of its heat would be removed by its contact with the subannular mass.

In the winter hemisphere, no tropical air mass would be present, but a migrating line of storms would follow the approximate line of contact of the subannular air mass with the polar air. These storms would be least intense at the

solstices, for then the subannular air mass would be most extended, and the temperature difference between subannular and polar air least. As the shadow of the ring crept equatorward, the subannular air mass would contract, and would also receive decreasing amounts of radiation, while the polar air mass would extend into areas of greater insolation. Near the time of the equinoxes, a small, temporary, intermediate air mass, warm in relation to the subannular, might be created in the winter hemisphere between the polar and subannular masses, an occurrence that would tend to increase precipitation on the poleward edge of the subannular mass. Thus, in the winter hemisphere, the storms at the outer edge of the shadowed area would move poleward and become less intense from equinox to solstice; and migrate equatorward, increasing in intensity, from solstice to equinox.

This summary of atmospheric behavior, explaining, in a very general way, terrestrial climatic conditions during the assumed existence of a ring of minute satellites during early Permian time, might also serve as a rough guide to present-day Saturnian climatic conditions.

The foregoing descriptions, which have been greatly simplified for the sake of clarity, do not take into consideration the distribution of land and sea, a factor which necessitates the division of the present tropical and polar air masses into maritime, continental, and modified groups. Consideration of these omitted data would lengthen the descriptions to several volumes, and would modify the details, without adding materially to the general discussion.

With a ring of sufficient opacity, annual snowfall at and near the equator would exceed annual attrition, leading to the formation of glaciers in equatorial lands, while conditions were less favorable to their presence elsewhere. In lands not shaded by the ring, or shaded for only a small part of each year, climatic conditions would be definitely unfavorable to the formation or continuing existence of ice fields.

The shadow of the ring, with its effect on atmospheric circulation, is sufficient to account for most features of the early Permian ice age. Continued accumulation of snow at the equator would lead to the formation of glaciers there, and to poleward ice motion, which is in general accord with

the field evidence from India, South America and Africa. According to this hypothesis, the glacial phenomena evidenced by the Squantum Tillite[18] and the (disputed) Caney Shales[19,20] are normal, and not something to be explained away with difficulty.

Reference to Fig. 1 will show that during the early Permian more ice was present south of the equator than north of it. This apparent asymmetry of the glaciated areas about the equator is partly due to the incompleteness of present evidence, but the Tethys Sea of early Permian time, with its modifying effect on climates, would tend to reduce the size of ice fields north of the equator in Africa and India. The ancient Carribbean Sea would have a similar effect in the Americas.

During the Upper Carboniferous and Permian, according to evidence cited by Brooks,[43] the climate of parts of the Antarctic was "temperate to hot or cold desert. No definite evidence of glacial conditions, but strong evidence of seasonal climate." This is in accord with supposed conditions during the existence of the hypothetical ring, but decidedly embarrassing to both the continental drift and land bridge hypotheses. In contrast, according to evidence cited by Coleman,[43] ice in parts of Australia and in the Falkland Islands came from the south during the Permian ice age. This evidence supports the hypotheses of continental drift, and of land bridges, and opposes the ring hypothesis here presented. It is suggested that the northward-moving ice of Australia and the Falkland Islands, although of Permian or late Pennsylvanian age, was not coexistent with the poleward-moving ice of Africa, India, and South America. It is apparent that the evidence of this ice motion is contradictory to that of nonglacial climates in parts of Antarctica, and that some of the evidence disagrees with any hypothesis yet presented to explain the climates of the early Permian.

Destruction of the Ring.

Any ring of minute satellites in orbits close to their primary will eventually be destroyed by the shrinkage of the orbits of the components under tidal influences. Mathematical consideration of the rate of shrinkage suggests that were the

only destroying forces tidal a ring of satellites formed about earth during the early Permian would still be present.

Other, and probably more quickly destructive, forces exist. Collisions between the component fragments, unless the particles are perfectly elastic, will transform some of the energy of the system into heat, which will be lost by radiation into space. In a ring system, where the component particles occupy rather closely adjacent orbits, collisions would seldom be single encounters. Each party to an initial collision, moved slightly from its normal orbit, would tend to carom off through the system, and to engage in further energy-dissipating encounters. A further result of these collisions would be increased fragmentation of the particles.

Another part of the system's energy would be dissipated as a result of friction between the satellites composing the ring and the highly tenuous outer atmosphere of the primary. This loss will be slow, for the outer atmosphere is not dense, but during the life of such a ring, the energy loss, although very gradual, should be appreciable. Calculations [44] suggest that the pressure of the earth's atmosphere, at the radius of limiting stability for a satellite as dense as the earth, is of the order of 760×2^{-1917} mm. of mercury. All other factors being the same, this "air friction" will tend to have a maximum retarding effect on the smallest particles, as is indicated by the well-known square-cube relation. The ring's components, however, are not spherical, but fragments having various shapes. It seems probable that the "air friction" would be considerably in excess of purely theoretical values, for any irregularly-shaped body moving through a resisting medium tends to so orient itself that maximum resistance is produced, as is demonstrated by the attitudes of falling snowflakes. The possible effects of a resisting medium on the motion of satellites are discussed in some detail by Jeffreys.[45]

The time necessary for the destruction of a ring of a given size is unknown, and cannot be calculated with any assurance from data now available. In the Saturnian ring system, which has been known since the time of Galileo, slight changes have been observed in 300 years. Hence, it may be assumed with some confidence that a comparable ring about earth would last long enough to permit the formation of great ice

sheets under it, but that, unless rejuvenated by the addition of more material, its life would be measured in tens of thousands, rather than in millions, of years. A rather complete discussion of the factors leading to the destruction of a ring system is given by Maxwell.[46]

Multiple Permian Glaciations.

Geologists are in general agreement that there were several circumequatorial glaciations during the early Permian, with relatively warm interglacial intervals separating them. These, obviously, could hardly have been produced by a single ring about earth, the effects of which gradually decreased with age.

Several explanations of this multiple early Permian circumequatorial glaciation, all in accord with the ring hypothesis, are possible. The simplest is that more than one small satellite disintegrated and formed a ring about earth. The disintegration of each satellite, according to this explanation, just preceded a glacial interval; the disintegration of each ring led to an interglacial period. The sequent destruction of several nearly co-orbital satellites, while not impossible, seems hardly probable.

A more reasonable explanation, requiring the disruption of only one small satellite, is suggested by Jeans' description of the fate of a noncompressed satellite when it comes within the Roche Limit. According to Jeans,[35] after the initial breakup, some of the satellite's fragments would not suffer further disintegration at first, while others, being entirely within the limit, would be continually subject to disrupting tidal forces, and would form a ring of small fragments in orbits about their primary. Later, the hitherto untouched fragments, having orbits at, or just outside of, the Roche Limit, would be drawn inward and disrupted, their fragments mixing with those already parts of the ring, thereby rejuvenating it.

This process might well account for the alternate cold and warm climates of the early Permian. After the ring was formed, causing circumequatorial glaciation on earth, its component particles would gradually drop out, decreasing the opacity, and permitting warmer equatorial climates.

This would lead to a glacial recession or an interglacial interval. Later, a previously independent fragment would creep inward, as a result of tidal forces, be disrupted, its fragments join those of the ring, increasing the opacity, and another glacial interval would result. This process would repeat itself until no more fragments were available, after which the final dissolution of the ring would take place.

In addition to possible variations in the opacity of the ring, various changes might take place in the intensity of solar radiation, in the transmission characteristics of the atmosphere, and in the topography of the earth. All of these would, in one way or another, affect the ice fields, and until all of these changes are determined, no valid speculation as to the details of the life of the hypothetical ring can be made.

Possible Future Tests of Hypothesis.

From the evidence given and cited above, it seems probable that there was no ring about Saturn when *Brachydactylopus fontis* hunted for food on the muddy shore of the Fountain Sea, and that there will be no such ring when the advanced beings of the far future dig up fossilized remains of the primitive men of the fourth Pleistocene interglacial. Rings, from their very nature, are astronomically short-lived.

The ring hypothesis, here advanced to explain the peculiar climates of the early Permian, is based on the premise that a ring, similar to the Saturnian ring, formed about earth during the early Permian, and vanished without leaving other trace than that of these peculiar climates.

In the future, according to the reasoning here followed, Saturn's ring should slowly creep inward and disintegrate. Some scanty evidence of inward motion has already been collected in the 300 years that the system has been observable. Two minor satellites in the solar system are now very close to the Roche Limit—Phobos, inner moon of Mars, and Jupiter's unnamed innermost satellite.[47] These should eventually suffer the same fate as hypothetical Ephemeron, but their destruction, as least in the case of Phobos, which revolves about an arid planet, may be long delayed as a result of their small mass and the small amount of tidal friction on their primaries. When, and if, Phobos disintegrates, climatic

changes on Mars, produced by the shading effect of the ring formed from the satellite's fragments, should be observable, and might resemble those on earth during the early Permian. Lack of water would certainly prevent the formation of extensive ice fields, but a thin band of snow extending for a short distance on each side of the Martian equator, the closest approach to glaciation possible there, might be produced, and if present could certainly be seen by observers on earth.

The time of destruction of Phobos and of Jupiter's innermost moon cannot be determined with any assurance, for neither the densities of these diminutive satellites, nor surface conditions on their primaries, are known with any accuracy. It seems quite probable that Jupiter's moon will be the first to go, as it is closer to its primary, and a dense atmosphere exists there.

In the very distant future, as (and if) the solar system "runs down," earth may capture some wandering mass of cosmic junk and again acquire a ring like that postulated to explain the Permian glaciations. Speculation concerning the time of capture of this as yet unknown body is futile, for exact data, or even sufficiently detailed hypotheses, upon which to base the reasoning, are not now in existence.

According to a number of theories, notably those of Jeffreys and Darwin, the moon will ultimately be drawn toward earth by tidal forces, and upon reaching the Roche Limit will be broken up, the fragments forming a ring of small satellites about earth. This ring will probably produce such changes on earth that life in forms resembling the present will be impossible. Calculations suggest that this cataclysm will take place in about eight billion years, an interval about three times that from "creation" to the present.

Summary.

The ring hypothesis here outlined, with its almost embarrassing number of alternatives, attributes the Permian ice age to the shading effect of a ring of minute satellites, all having orbits about the earth within the Roche Limit. The postulated source of the ring's material was a hypothetical satellite, Ephemeron, which came within the Roche Limit just prior to the Permian ice age, and was torn apart by tidal

forces. Repeated glaciation during the early Permian is attributed to the thinning of the ring by loss of component particles, and to its rejuvenation by the disintegration of large fragments of the original satellite which came within the "danger zone" after the initial disruption.

This explanation seems to account for all the major features of the early Permian, and to be in general accord with current astronomical theories.

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The writer is indebted to a number of geologists, physicists and astronomers for helpful suggestions and discussions in connection with this study, and to Dr. O. C. Lester, of the University of Colorado, for a critical reading of this manuscript.

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$$R = 2.4554 \sqrt[3]{\frac{D}{d}}\, r,$$

where R is the Roche limit, measured from the planet's center,
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D is the density of the planet,
d is the density of the satellite.

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36. Very little is known concerning the exact chemical composition of the various sedimentary beds. Lest the Permian red beds be cited as evidence that Ephemeron was made of iron, it should be noted that these red beds are evidences of desert conditions, and that the Permian deserts were no more extensive than those of the present day, where the red beds of the far future are now forming.
37. Studies of the aurora borealis, by Sturmer (HUMPHREYS, W. J., *op. cit.* (2), p. 18), show that a highly tenuous atmosphere exists as far as 500 miles above the earth's surface. Assuming that the atmospheric pressure halves for each three mile increase of elevation, the pressure at the height of the aurora would be 760×2^{-166} mm. of mercury.
38. As the particles forming a ring become quite small, and their average size

approaches that of a grain of sand, the effects of rapid thermal change, collisions, and tidal stresses become negligible, and the actual number of collisions may be greatly changed as a result of static electrical charges carried by the particles. So little is known of either the manner of production of these charges or their magnitudes that no prediction of their effect on the life of the ring can be made. It seems probable, however, that static electricity in considerable amounts would be produced, and that as a result the auroral displays would be disturbed. Investigation of a possible correlation between mechanical disturbances in the Saturnian ring system and electrical disturbances on earth might shed considerable light on the problems of "background" noises and of the properties of the Kennelly-Heaviside layer.

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THE BREAKING THEORY OF DETONATION.

BY

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INTRODUCTION AND PRELIMINARY DISCUSSION.

An attempt will be made in this paper to define the nature of the type of decomposition, known as detonation, as it occurs in solid and liquid materials. Before such an attempt, however, it is necessary to clear the ground; first, by defining "explosion" and differentiating between explosion and the cause of an explosion; second, by showing the untenability of presently accepted theory; third, by showing that present methods of testing, the characteristics of explosives, show only the most empirical results, that is, no test can be correlated with any other test, and fourth, to present data and evidence supporting a new and more rational theory.

An explosion results when any material suddenly increases in volume. It is not necessary to describe any preceding event. The causes of explosions are many. It may be the failure of a gas cylinder; the sudden increase in the temperature of a material with production of a gas; a rapid chemical reaction with the production of gas; or it may be the breaking down of a high explosive compound; the nature of which is the subject of this paper.

There are two types of materials which are known as explosives, one of these, the earliest known, black powder, causes explosion by ordinary rapid combustion,* for this reason there has been a tendency to explain the explosions of the other type as a similar reaction of greater speed.[1] But it is known that the rapidity of the reaction of black powder is dependent on the intimate contact of the three ingredients, or in other words the fuels and the oxidizing agent. On the other hand

* Some exceptions to this will be made later in this paper.

[1] "However, the known facts do not in my opinion oblige us to admit of a difference in nature or of a sharp line of demarcation between the two varieties of phenomena, i.e., burning and detonation." From "Explosive Materials," M. Berthelot, D. Van Nostrand, 1883, N. Y.

the high explosives, such as tri-nitro glycerine are pure compounds and cannot be modified to produce more intimate contact between the fuels and the oxidizing agents, by finer division.

The high explosive compounds have been known for many years but not as such. Picric acid was known for approximately 100 years before it was suspected that it was a powerful explosive. This fact further emphasizes the distinction between such mixtures as black powder and the true high explosives. For black powder simple inflammation is sufficient to initiate the release of its energy. On the other hand the high explosives require a very special application of energy to release their energy in the violent manner of detonation. When simply ignited most of them burn quietly, some of the more insensitive ones barely maintaining combustion, and burning with a smokey flame, due to their deficiency in oxygen. The application of the most intense heat is not effective to bring about detonation in some of the more insensitive explosives.* Yet when properly iniatited, they produce an explosion distinctly more violent and shattering than that produced by such mixtures as black powder, which reacts at the interface only. The mixtures excepting the gases burn at a higher rate the greater the area of contact between the reacting materials. The mixtures also burn more rapidly at low densities. The pure high explosives, on the other hand, are homogeneous compounds, in which the relation between the reacting ingredients cannot be changed. This is another characteristic which distinguishes combustion proper from detonation, which is propagated at a higher rate the higher the density, of the material. Higher atmospheric pressure also causes higher burning rates for the mixtures, but this is not effective when

* Whereas black powder is practically insensitive to percussion it is easily exploded by the agency of a flame, whereas with the organic nitrates the exact opposite holds good. Indeed nitroglycerine could be ignited and allowed to burn with the development of a flame without any fear of explosion; but a blow or slight percussion gave rise to an explosion of considerable intensity. Looking back on this period it is obvious that a considerable number of organic products which we know now as extremely powerful explosives were in existence, but were not even taken into consideration in this respect prior to Nobel's discovery (English Patent No. 1345, May 7th., 1867) owing to their stability and to the insufficient means to hand by which they might be ignited." From "High Explosives," Colver, 1918, pages 3 and 4, The Technical Press, London.

applied to detonation. According to Stettbacher, Kast found that T.N.T. detonates in vacuum at a rate equal to the rate in air or other gases. This phenomenon, known as detonation progresses through the material at rates of from 2000 to 8000 meters per second, approximately, depending on the nature of the material. The high explosives are compounds which are inherently unstable. In discussions of the characteristic inertia and instability of carbon compounds Nernst [2] characterizes such compounds as follows; "This same inertia renders it possible to build up molecules whose arrangement, so to speak, have an excessive complexity and accordingly is 'Unnaturlick.' Through a more or less energetic blow there follows a transformation into a more stable arrangement of the atoms. Usually in this change a large amount of energy is set free, and the result becomes an explosion; so it becomes clear why such a large number of explosive substances are found among organic chemicals."

Many theories have been advanced in efforts to define detonation, among which is that of Berthelot's, which has received the most consideration and therefore his theory will be discussed at some length, and an attempt be made to show the most pertinent points of his theory by quoting from his translated work.[3] The following is quoted from page 36, "The author is of opinion *that every explosive reaction should be attributed to a preliminary heating*, which is gradually transmitted, directly or indirectly, raising successively all the parts of the matter to the temperature of decomposition, shock, pressure, friction or mechanical effects are only efficacious by causing this preliminary heating. . . ." This is the most definite statement and is therefore given great weight. This statement indicates that Berthelot considers temperature as the immediate cause of every explosion. And from page 88 "The study of the various modes of decomposition of explosive substances and especially that of detonation as compared with combustion, and of explosion by influence, leads to the *admission* of the existence of a wave motion peculiar to and characteristic of explosive phenomena; this is the explosive wave. It will be more accurately and completely defined by showing

[2] "Theoretische Chemie," Nernst, 1926, Writer's Translation, page 339, Ferdinand Enke, Stuttgart.

[3] "Explosives and their Power," Berthelot, 1892, J. Murray, London.

how it is propagated in gaseous systems." And here it is indicated that Berthelot considers the detonation of all materials as similar to that of mixtures of gases. On pages 533 and 534 under conclusions is found the following: "It is in gaseous media that the study of it (the explosive wave) is at one and the same time the easiest and the strictest, and it is then that the results it offers are most far reaching, theoretically speaking. This study enables us, in fact, to show the existence of a kind of undulatory movement of a compound order, that is to say, produced in virtue of a certain concord of physical and chemical impulses within a substance under transformation. In the *sonorous wave* [4] the energy is weak, the excess of pressure stands at a minimum, and the velocity is *determined* by the mere *physical constitution* of the vibrating *medium*. On the other hand, it is the change in the chemical constitution which propagates itself in the explosive wave, and which communicates to the system an enormous energy and considerable excess of pressure. Like phenomena may become developed both in *solids* and in *liquids*."

It is desired to call attention to three points which are definite in these quotations; 1st., that in every case of explosion Berthelot believes there must be a *preliminary heating*, which is the means of initiating and transmitting the detonation, and 2nd., that he considers that the same mechanism is involved in detonation of gases, liquids, and solids, and 3rd., that the velocity is not thought to be determined by the physical nature of the material. All three of these points the writer hopes to prove to be definitely untrue. To quote from Nernst,[5] "It has thus been fairly established by experiment that the velocity of explosion of gaseous systems is a characteristic of the system, in the same way as, and in fact in close relation to, the velocity of sound in a gas. In the case of explosives in other states of aggregation however, the question still remains open; Berthelot [6] endeavored to determine whether liquid and solid explosives show a similar behavior to explosive mixtures of gases." And again from the same source; "The theory of solid and liquid explosives will prob-

[4] "Abel's Theory," *Phil. Trans.*, **159**, 67 (1869).

[5] "Theoretical Chemistry," 5th English Ed., Macmillan and Co. Ltd., London, pages 788–790.

[6] *Compt. Rend.*, 112, 16. *Ann. Chim. Phys.* (6), **23**, 485 (1891).

ably progress along the same lines as that of gaseous explosives; in particular, the pressure necessary to produce the explosion will be of fundamental importance for the *theory of the production* of the *explosion wave*. It must, however, be observed that, contrary to opinions sometimes expressed, a high temperature such as would initiate the reaction will certainly not be obtained by compression as is the case with gases." It is evident by these statements that this authority does not subscribe to the similarity of the mechanism of detonation of gases, to that of the liquids and solids.

A consideration of the outstanding differences between gases, and the liquids and solids should be sufficient, that is, the great compressibility of gases as compared to the slight compressibility of liquids and solids. Another outstanding difference is the fact that solids have rigidity of shape and even liquids have an instantaneous rigidity of shape. A further argument against Berthelot's "Preliminary Heating" theory of the initiation and propagation of detonation is the relation between the ignition temperature and the sensitiveness of explosives to other means of initiation. If the preliminary heating theory were true it would seem logical to expect that a low ignition temperature would indicate high sensitivity to the other means of initiation and be accompanied by a high rate of detonation. Data given in other parts of this paper will show that neither of these conditions is true. In fact some of the most sensitive explosives, such as mercury fulminate have comparatively low rates of detonation, and materials of the same sensitivity to heat have very different sensitivities to other means of initiation. Kast [7] found that gun cotton containing 20 per cent. water detonated at a rate of 6300 m./sec. as compared to only 1000 to 1800 m./sec. for the nitrocellulose powders which are chemically the same but are gelatinized and physically changed. Here the presence of a large percentage of water should exert a tremendous resistance to any tendency to reach the ignition temperature, which is a prerequisite to detonation according to Berthelot's theory. It is worth while, also, to note the great effect of physical condition; a much greater effect than is produced by extreme differences in chemical composition. Similar effects are noticed in gelatin

[7] "Zeit. f. d. Gesamte S. u S.," 1920, page 195.

dynamites, which after storage, lose their tendency to detonate at a high rate.

Examples of this effect of physical condition are shown by Coates and Perrott [8] in tests of gelatin dynamites, containing nitro-glycerine and ethylene glycol dinitrate. These materials were fired after various periods of storage, both in the confined and the unconfined condition, with and without priming charges of straight dynamite, yet, in almost every case, the rate of detonation, after storage was definitely much less than that of the original samples. Here we have chemically identical material, showing different rates of detonation, because of a physical change.

Berthelot [9] extends his theory to cover the transmission of detonation by influence, that is, transmission through an air gap from one charge to another, of similar explosive. Quotations are included, on this subject, from pages 87 and 533; "The explosive substance does not detonate because it transmits the movement, but on the contrary, because it stops it, and because it transforms its mechanical energy on the spot into calorific energy capable of suddenly raising the temperature of the substance up to a degree which causes its decomposition." And, "According to the new theory, explosive matter detonates by influence, not because it transmits the initial vibratory movement by vibrating in unison, but on the contrary, because it stops it and appropriates to itself the energy thereof." It would seem, then, that Berthelot believes the same mechanism is involved, in the initiation of detonation by influence, as that to which he ascribes the transmission through any other explosive, however initiated. Being initiated by the same agency, one would expect the wave to travel at the same rate in the explosive initiated by influence as when initiated by a detonator. This, however, is not true. The writer has found that a straight nitro-glycerine dynamite, when initiated by influence by a charge of the same material through a 10 inch gap, has a rate of detonation, approximately, 12 per cent.

[8] Reports of Investigation, Serial 2923, April 1929, Dept. of Commerce, Bureau of Mines.

[9] "Explosives and their Power," Berthelot, J. Murray, London, 1892.

lower than when initiated by a detonator.* Continuing the consideration of this theory, which requires that the vibratory movement be stopped and transformed into calorific energy, one would expect that the more resistant the material the more quickly the movement would be stopped, and therefore, the quicker and more effective the release of calorific energy. However, it is well known that explosives which have been pressed to a high density are more insensitive to detonation by a detonator or by influence, than the same materials, when in a loose condition.

A large part of the following discussion will be concerned with sensitiveness, because it is believed that sensitiveness may be called the "sine qua non" of explosive materials. It is the measure which differentiates explosives from all other materials. If sensitiveness is too great the elements cannot exist as a compound and if sensitiveness is too small the compound is not an explosive.

In Table I [10] are given the temperatures at which a number

TABLE I.

Explosive.	Explosion Point °C.
Guncotton (13 per cent. N)	183–186
Collodion cotton (12 per cent. N)	186–190
Guncotton powder, gelatinized fine grain	173–176
Nitro-glycerine	160–220
Explosive gelatin 7 per cent. collodion cotton	180–200
Nitro-glycerine powder (40 per cent. N.G.)	170–180
Dynamite (75 per cent. N.G.)	180–200
Picric acid	225+
Nitromannite	160–170
Nitro starch	170–175
Mercury fulminate	160–165
Black powder	225+
Ammonium nitrate powder	225+

of explosive substances decompose. The determinations were made by dropping one tenth gram of the substance into a

* This fact makes many efforts, at determining the rate of the impulse through a gap, unsuccessful. This applies especially to the work by G. St. J. Perrott and D. B. Gawthrop reported in THE JOURNAL OF THE FRANKLIN INSTITUTE, March 1927, pages 387–406.

[10] "Handbuch der Angewandten Physikalischen Chemie," Vol., 10, 1908, Barth, Leipsig, page 28.

loosely corked test tube immersed in an oil bath at 100° C. and raising the temperature at the rate of 5° C. per minute until detonation occurs. Although the materials are very diverse and have greatly different rates of detonation and sensitivity (which will become more clear later in this paper) nevertheless, the temperatures of ignition do not differ greatly.

Table 2 from Marshall [11] shows determinations made by

TABLE 2.

Composition.	500 Gm. Wt. Mm. Fall.	Temp. of Ignition °C.
Mercury fulminate	75–100	200
Silver fulminate	140	200
Sodium fulminate	over 320	150
Mercuric azide crystals .6–.9 mm.	65	200–210
Mercuric azide crystals 3 mm.	0	200–210
Mercurous azide	200	300
Silver azide	310	200
Cupric azide	0	200–210
Cuprous azide	260	200–210
Lead azide (finely divided)	260	340–350
Lead azide (moderate crystals)	130	340–350
Basic lead azide	280	380
Basic cupric azide	315	245

Wohler. In this table the sensitiveness to a falling weight is compared to sensitiveness to heat. An examination of the table fails to disclose any direct relationship. It would seem logical to expect greater sensitiveness for those compounds which have a low temperature of ignition, yet such a relation has not been shown. It seems unnecessary, therefore, to go to greater length to show the inadequacy of the preliminary heating theory.

It is now proposed to review briefly a number of efforts to define and measure the sensitiveness of high explosives, or the effectiveness of various agents to bring about detonation.

The results of a more recent and complete investigation given in Table 3 is taken from work by Taylor and Rinkenbach [12] and shows only the most remote or no relation between sensitiveness to heat and to impact or friction. In fact five

[11] "Explosives," Marshall, Vol. II, 2nd. Ed., pages 509–510, J. & A. Churchill Ltd., London.

[12] JOURNAL OF THE FRANKLIN INSTITUTE, Sept. 1927, pages 369–376.

TABLE 3.

Explosive.	Pendulum Friction.			Impact Fall Cm.	Explosion Temp. °C.
	Added Wt. Kg.	Fall Cm.	Swings No.		
Silver acetylide C	0.0	12.5	2	66	171
Cyanuric triazide	0.0	12.5	3	7	252
Fulminate-chlorate 90–10	0.0	25	3–4	23	240
Fulminate-chlorate 80–20	0.0	25	3–5	19	235
Mercury fulminate	0.0	25	3–10	24	260
Silver fulminate	0.0	25	10	38	245
Silver acetylide B	0.0	33	4	15	177
Silver tri-nitro resorcinate	0.0	33	7	28	305
H.M.T.D.	0.2	37.5	5–7	8	200
Silver acetylide D	0.2	50	8	79	200
Lead azide	0.45	37.5	12	43	383
Dulcitol hexanitrate	1.0	25	14	47	205
Silver acetylide A	1.0	33	12	43	225
Mercurous azide	1.0	50	16–17	6	298
Lead picrate	1.0	50	23–37	4	281
Lead tri-nitro resorcinate	1.0	50	29–37	50	293
Silver acetylide F	2.0	50	16	64	200
Silver azide	4.35	33	39	41	273
Silver picrate	>4.35	50	27–38	34	335
Barium picrate	>4.35	50	33–36	>110	403
Copper picrate	>4.35	50	34–40	>110	373
Mannitol hexanitrate	>4.35	50	49–54	24	232

different samples of the same material produced by slightly different methods vary as much as the most extreme variation between any of the seventeen remaining materials. The material referred to is silver acetylide. In Table 4 the relative

TABLE 4.

Explosive.	Pendulum Friction.		Impact Fall Cm.	Explosion Temp. °C.
	Added Wt. Kg.	Fall Cm.		
Silver acetylide A	1.0 (15)	33	43 (15)	225 (7)
Silver acetylide B	0 (9)	33	15 (5)	177 (2)
Silver acetylide C	0 (1)	12.5	66 (19)	171 (1)
Silver acetylide D	0.2 (12)	50.0	79 (21)	200 (3)
Silver acetylide F	2.0 (19)	50.0	64 (18)	200 (3)

The comparative position of each type of the material is given in parenthesis in the Table.

sensitiveness of each variety of this material is indicated by numerals; the lowest numeral in parenthesis indicating the

highest sensitivity. Thus it is shown, that different samples of one material range in sensitivity all the way from first to twenty-first by these tests. The maximum variation possible is from first to twenty-second. Even the ignition temperature which according to the authors is easily reproduced, varies a total of 54° C. Yet the different varieties of this compound are so similar that they can only be differentiated by giving the details of the method of production. One variety of this same compound is as sensitive to the friction test as cyanuric triazide and more sensitive to heat. Lead picrate appears to be more sensitive to impact than cyanuric triazide. Now neither lead picrate nor silver acetylide are considered dangerous in comparison with commercial primary detonating compounds, yet the authors say [13] that cyanuric triazide is too sensitive to handle commercially.

The value of the results are diminished, by the fact that such minor changes, in the details of the equipment or procedure have such a great effect upon the results. Taylor and Rinkenbach show that the difference between the use of hard steel and ordinary steel, in the impact machine may cause differences, in the results amounting to 100 per cent. or more. A slightly roughened surface on the face of the impact block, caused very erratic results. Lead picrate required a drop of only 4 cm. when the plunger was pressed *firmly* on the material and a drop of 36 cm. when only compressed by the weight of the plunger. This would seem to indicate that not only does temperature of explosion have little relation to sensitiveness to detonation, but that any present method of determining sensitiveness as a fundamental characteristic has very little value. In fact it would be easy to find compounds which have a very low ignition temperature, which cannot be detonated at any test. It is well known that many materials which are not explosives, have ignition temperatures as low as many of the explosive compounds. The following Table 5 [14]

[13] Reports of Investigations, Dept. of the Interior, Bureau of Mines, Serial 2513, Aug. 1923.

[14] "Fuels and their Combustion," Haslam and Russell, 1926, McGraw-Hill Book Co., N. Y.

TABLE 5.

Material.	Ignition Temp. C.
Sugar	385
Pennsylvania crude	367
Gas oil	336
Kerosene	295
Acetaldehyde	185
Benzaldehyde	180

gives the ignition temperatures of some well known materials.

These temperatures are all within the range of the explosion temperatures of the explosive compounds. It is not the purpose to discredit the tests which have been discussed, for such tests have their value, from a practical point of view, to show what different rough handling the particular material, being studied, will withstand. Nevertheless, the tests are not to be taken very seriously, even from the practical point of view, when it is considered, that successive batches of the same compound, not distinguishable chemically show divergent characteristics by the same or different tests. The results to this point would, therefore seem to be sufficient to show that no relation exists between explosion temperature and sensitiveness to other means of initiation of detonation.

Continuing the review of sensitiveness tests Table 6 taken from Marshall [15] of work by Bichel [16] and Brunswig [17] gives the results of drop tests in which weights ranging from 0.1 to 20.0 kilograms have been used. That is, the same explosive has been tested for sensitiveness by using different weights in a drop test. In almost all other reported work of this nature no attempt has been made to determine whether or not a certain weight would give the most representative results, but one weight has been chosen arbitrarily and the results taken as representing something definite. This series of tests, however, tend to show that the relative sensitiveness of a number of explosives will depend upon the particular mass chosen. For instance, Nos. 7 and 8 in the table show no difference when tested with the 0.5 kg. weight, but No. 7 requires double the height of drop when tested with the 0.1 kg. weight. Also the

[15] Marshall, "Explosives," 1917, J. & A. Churchill Ltd., London.

[16] Bichel, "Marine-Rundshau," 1905, page 1345.

[17] Brunswig, "Explosivestoffe," 1907.

Nos. 7 and 9 are similar when tested with weights below 1.0 kg. but are very dissimilar with the 1.0 and 2.0 kg. weights. A similar condition holds in regard to Nos. 10 and 11 so extremely different when tested with the 0.1 kg. weight yet not distinguishable by weights above 1.0 kg. Regardless of the height required by any explosive in this list at weights less than 20.0 kg. none of them can be distinguished by the use of the 20.0 kg. weight. These results indicate that one method for such tests must be selected if a comparison is to be made, yet that method may or may not be the best method, and also that the total energy exerted may not be as important as the time energy relation.

TABLE 6.

Explosive.	Fall in Cm. Required Falling Wt. in Kg.						
	0.1.	0.25.	0.5.	1.0.	2.0.	5.0.	20.0.
1. Nitrogen iodide.....	(b)5						
2. Mercury fulminate...	5–10	(b)5					
3. Nitroglycerine.....	5–10	(b)5					
4. Guhr dynamite.....	5–10	(b)5					
5. Gelatine dynamite....	5–10	(b)5					
6. Dinitro glycerine.....	10–15	5–10	(b)5				
7. Gel. dynamite from it.	30–35	15–20	5–10	(b)5			
8. $KClO_3$ + S.....	15–20	10–15	5–10	(b)5			
9. Gun cotton, dry.....	30–40	15–20	5–10	5–10	5–10	(b)5	
10. Flake powder.....	40–50	20–30	10–15	10–15	10–15	5–10	(b)5
11. Gun cotton, wet 15% H_2O.....	a200	70–80	15–20	10–15	10–15	5–10	(b)5
12. Black powder.....		a200	190–200	100–110	30–40	5–10	(b)5
13. Picric acid crystallized			a200	190–200	100–110	50–60	(b)5
14. Picric acid compressed				a200	140–150	80–90	(b)5
15. T.N.T. crystalline....			a200	180–190	90–100	50–60	(b)5
16. T.N.T. compressed...				a200	150–160	80–90	(b)5
17. Gelatine carbonite....	80–90	20–30	5–10	5–10	5–10	(b)5	
18. Kohlen carbonite.....	a200	50–60	20–30	20–30	20–30	5–10	(b)5

The writer recalls many instances in which an explosive failed to explode on the first impact but fired when the weight impacted after rebound. Such an occurrence must be due to a sort of resonance, for surely the second impact was not as great as the first. The foregoing discussion confirms the conclusions reached by Taylor and Rinkenbach in regard to the drop test, with the added evidence that not only must the refinement and detail be checked with great care but that

even then, the results secured can only be duplicated by exactly the same equipment. Nevertheless a different method of testing, which has as much claim to consideration may arrive at a conflicting verdict.

We will now discuss, briefly, another method by which a figure can be secured for the sensitiveness of an explosive compound. The method referred to is that of determining the amount of an initiating compound which will cause detonation of another less sensitive compound. In Table 7 and 8 are

TABLE 7.

Secondary Explosive.	Mercury Fulminate.	Ful-Chlorate 90–10.	Ful-Chlorate 80–20.
	Initiator required for complete detonation, grams.		
T.N.T.	> 1.00	> 1.00	> 1.00
T.N.A.	.45	.31	.31
Picric Acid	.40	.37	.37
Tetryl	.35	.30	.27

TABLE 8.

Secondary Explosive.	Mercury Fulminate.	Ful-Chlorate 90–10.	Ful-Chlorate 80–20.
	Initiator required in grams.		
T.N.T.	.26	.25	.24
T.N.A.	.20	.17	.17
Picric acid	.25	.23	.22
Tetryl	.24	—	—

given results reported by Storm and Cope.[18] In these tables the secondary explosives are the less sensitive.

In the tests of Table 7 0.4 gm. of the secondary explosives were used in No. 6 blasting cap cups and pressed slightly. The initiating charge was consolidated slightly by tapping on a hard surface. No reënforcing cups were used.

In the results reported in Table 8 both the secondary explosives and the primary detonating explosives were pressed at 3000 lbs. per sq. inch and reënforcing cups were used. In the first series of tests, Table 7, T.N.T. appears distinctly less

[18] Technical Paper No. 125, Dept. of the Interior, Bureau of Mines.

sensitive than the three other compounds. Tetryl is indicated as the most sensitive in this series. In the second series of tests, Table 8, T.N.T. is only distinguishable from the others by one or two hundredths of a gram of the initiating explosive, and T.N.A. is indicated as distinctly the most sensitive.

Table 9 is built of two Tables given by Taylor and Rinkenbach.[19]

TABLE 9.

Secondary Explosive.	Cyanuric Triazide.	Mercury Fulminate.	Lead Trinitro-Resorcinate.
	Initiator required in grams.		
Tetryl	.04	.20	.80+
Picric acid	.05	.21	.75
T.N.A.	.09	.20	
T.N.T.	.10	.26	1.00+
Trinitro-resorcinate		.20	1.00+
Amm. picrate	.15	.85	
Guanidine picrate		.30	1.00+

These tests were conducted by weighting out 0.4 gm. of the secondary explosive and then adding the initiator and a reenforcing cup and pressing at about 2800 lbs. per sq. in. In this series picric acid seems to be the most sensitive. But the only general statement that can be applied to the results is that there is no unanimity as to the relative sensitiveness of any of the materials. By the three series of similar tests three different compounds are indicated as the most sensitive.

Then, if we examine the results from the angle of the effectiveness of the initiators, more inconsistencies appear. In Table 7 T.N.A. requires a greater amount of fulminate than does picric acid, but the reverse is true for both the 90–10 and the 80–20 fulminate-chlorate mixtures. This is quite surprising, when the great similarity of these initiators is considered. In Table 9 T.N.A. requires two and one-quarter times as much cyanuric triazide as is required for tetryl, but both of these secondary explosives require the same amount of fulminate. Ammonium picrate requires only one and one-half times as much cyanuric triazide as does T.N.T., whereas it

[19] Reports of Investigation No. 2513, Aug. 1923 and 2533, Oct. 1923, U. S. Bureau of Mines, Dept. of the Interior.

requires three and one-quarter times as much mercury fulminate as does T.N.T., and finally picric acid requires more of either cyanuric triazide or mercury fulminate, than does tetryl, but requires less of lead trinitro-resorcinate. Such tests are very interesting, because they tell us that there must be many factors entering into the sum total of sensitiveness and effectiveness of these explosives, which are totally unknown. The initiators cannot be classified as being more or less effective, in general, because they are somewhat selectively effective, and the secondary explosives are selectively susceptible to the initiators. At least, there must be an equation in which the result is the effect of the reaction of a number of factors. It is quite probable that we have a case of the time energy relation and effect of synchronism. Certainly, we should not expect the preliminary heating theory to clarify such irregularities.

A number of rash statements have been made in regard to the sensitiveness and rate of detonation of high explosives, and the relation these qualities bear to certain other qualities. Most of such statements are mere guess work as is the following from Barnett:[20] "By the violence or brisance of an explosive is meant the shattering effect produced. This is rather an elusive quality and depends not only on the power but also on the maximum pressure developed, this in turn depending on the velocity of detonation." Here three terms are used to describe one quality. That quality is said to depend upon three other characteristics but no attempt is made to define the relationship. Gawthrope[21] says: "Other things being equal, the higher the rate of detonation the greater will be the shattering effect of the explosive." But other things cannot be equal. That would require that two explosives be identical except in rate of detonation. And how shall we define shattering effect? The sand test has been used in attempts to give it a numerical value, but actually only shows how much sand of a specified type will be crushed. This does not give any information as to the effectiveness in crushing other materials.

[20] "Explosives," by DeBarry Barnett, 1919, page 184, Bailliere Tendall and Cox, London.

[21] JOURNAL OF THE FRANKLIN INSTITUTE, Jan. 1927.

It is believed that shattering effect or brisance should be considered as a qualitative rather than a quantitative property.

Taylor and Monroe [22] found that with detonator cups having thicker walls and bottoms a slightly greater amount of sand was crushed. These results agree with the experience of the writer. It can be assumed that a greater amount of metal will slow down the shock delivered by the explsovie and yet the amount of sand crushed is increased. This would seem to indicate that the amount of sand crushed does not measure the quickness of the explosive which is sometimes used as synonymous with brisance. In the same paper Taylor and Monroe reported results of tests with mercury fulminate in the sand test using loading pressures varying from zero to 10,500 lbs. per sq. in. These results show a gradual decrease in the amount of sand crushed from 39.4 grams at zero pressure to 32.1 at 10,200 lbs. per sq. in. and failure of the charge at a pressure of 10,500 lbs. per sq. in. It is known that at the higher pressure with concomitant higher densities a higher rate of detonation of the mercury fulminate will result. Still at the higher rate the fulminate is less effective to crush sand.

Another interesting point is brought out in regard to the relation of the rate of detonation to another characteristic of explosives, by the results secured by Coates and Perrott [23] in their tests of the effect of storage on gelatin dynamites. In their paper results of rates of detonation and strength tests are reported for gelatin dynamites before and after storage. The strength test employed is the ballistic pendulum. In this test the explosive is fired in a heavy steel cannon in the unprimed condition. That is, without the use of a priming charge of straight dynamite. Therefore a comparison of results before and after storage will be made of results using the rates of detonation in steel tubes, also unprimed. Such a comparison shows that the rates decreased approximately 33 per cent. on the average, after storage, and the strength as indicated by the ballistic pendulum decreased only 1.5 per cent. on the average. These results bear out the writer's

[22] Reports of Investigation, Bureau of Mines, Dept. of the Interior, Serial No. 2558, Dec. 1923.

[23] Reports of Investigation, Bureau of Mines, Dept. of Commerce, Serial No 2923, April, 1929.

opinion, that the rate of detonation has very little effect on, or relation to other characteristics of explosives, such as total power or shattering effect. Nevertheless, many still consider the rate of detonation equivalent to the rate of reaction. This is shown by the following quotation from Hayes,[24] "The rapidity of reaction of an explosive may be determined directly, by experiment, in high explosives. This is termed the rate of detonation of the explosive, and is a measure of its brisance." Quoting Burnswig,[25] "Berthelot comes to the conclusion that, other things being equal, the sensitiveness of explosive substances is the greater the more the heat developed by the explosive reaction." This statement is quite obviously untrue. In fact most of the very sensitive compounds are low in total energy; most of them contain a heavy metal as one component which precludes a high energy content. In fact there does not seem to be any two characteristics of any explosive which can be correlated. No property connotes any other property. To be specific a high rate of detonation does not guarantee a high total power or shattering effect, nor great sensitivity. Low ignition temperature does not indicate high sensitivity to other means of initiation.

A number of formulæ have been written to calculate the power or brisance, or some other quality of explosives. Kast has been responsible for many of these efforts. Table 10 is

TABLE 10.

Explosive.	Density.	Heat of Expl. Cal./Kg.	Expl. Temp. °C.	Gas Vol. l./Kg.	Spec. Energy.	Rate of Det.	Brisance Value.
Black powder	1.2	665	2,380	280	2,810	400	1,350
Ammonium nitrate	1.0	347	1,230	980	5,575	1,920	10,704
Ammonium perchlorate	1.17	312	1,410	763	4,860	2,500	14,216
Di-nitro-benzol	1.50	870	2,500	670	7,035	6,100	64,400
Nitro-cellulose	1.30	1,025	3,100	765	9,765	6,300	79,800
T.N.T.	1.59	950	2,820	690	8,080	6,700	86,100
Tetryl	1.63	1,090	3,370	710	9,790	7,200	114,900
Nitro glycerine	1.60	1,455	4,250	715	12,240	7,450	145,900
Blasting gelatine	1.63	1,540	4,300	710	12,285	7,800	156,300
Hexa-nitro-mannite	1.73	1,454	4,370	694	12,190	8,260	174,230
H.M.T.D.	1.70	1,365	3,400	908	12,619	8,380	179,790
P.E.T.N.	1.62	1,528	4,300	780	13,500	8,400	183,670

[24] "Elements of Ordnance," Lt. Col. Hayes, 1938, page 17, John Wiley and Sons, N. Y.

[25] "Explosives," Brunswig, Trans. by Munroe and Kibler, page 17, John Wiley and Sons, N. Y.

taken from "Chemiker-Zeitung" [26] and is a compilation of the results of work by Kast and his co-workers. The specific energies and the brisance values have been calculated by the use of such formulae. The significance of the results are not apparent. The table is translated for convenience. The headings should be self-explanatory.

The first three substances in this table are considered at very low densities and the remainder at densities near their maximum. The reason for such a procedure is not known. The rate of detonation which is taken as one of the factors in the formula, for the brisance value, varies with density, and varies in a different way for every explosive. We, therefore, have no right to compare explosives by any formula which uses the rate of detonation as a factor, unless the explosives are taken at the same relative density. Black powder should not be considered at all for the rate given is not a rate of detonation but a rate of burning, and if the density of this material were increased the rate of burning would be decreased, thereby *giving a lower value for a higher density* whereas, with a detonating explosive, the rate of detonation would be increased.* A third objection to the table is the fact that the temperature of explosion is only a guess. On the subject of explosion temperature Stettbacher [27] says, "The new calculations of the explosion temperature which have been made necessary by changes in the figures for the specific heat and amount of dissociation at high temperature still continue to maintain the old uncertainty on this subject. This uncertainty is made still more confusing by the occurrence of endothermic compounds such as H_2O_2 and dissociation of the diatomic gases, both of which have a tendency to reduce the temperature. . . . On the grounds of the theories of Planck and Einstein, Bjerrum has calculated the specific heat (for hydrogen) for constant volume between 1200° C. and 4000° C. Even these figures have not been entirely free from the attacks of the critics, and so it is that every calculator of the explosion temperature selects a different molecular heat and extrapolates the explosion

[26] "Chemiker-Zeitung," 1925, page 642.

* Black powder will detonate for a short distance but will not maintain detonation. Any material may be penetrated by a detonation wave but only the true high explosive will sustain the wave indefinitely.

temperature by which means different investigators arrive at temperatures which may differ by as much as 1000° C. For instance, Bichel gives 3200° C. for explosive gelatine, Kast gives 4300° C." Discussing Kast's work Stettbacher says, all such formulæ are mere attempts to fit figures to the practical results, "Because every explosive effect is the resultant of the most diverse forces and resistances, modified by accidental conditions which cannot be expressed by any fixed formula." And again speaking of the results secured by Kast on the rate of detonation of nitrocellulose, Stettbacher [27] concludes as follows, "That is to say: concerning the intimate process of explosion and more especially the mechanism of its propagation there is nothing but darkness and the most vague speculation."

As a last example of the inadequacy of our present knowledge of explosives there is the disastrous explosion at Oppau in 1921. It had been the custom to break-up the mixture of ammonium nitrate and ammonium sulphate, which caused the explosion by blasting. No one had considered the procedure dangerous. Nevertheless the material caused the most disastrous occurrence of its kind in history. After a long investigation [28] the only conclusions that were reached were that a physical or chemical change or both had taken place in the material which had caused a change in its explosibility. It is difficult to believe that in this age of science, one important branch of industry should continue to be dominated by a mystery.

It is hoped that in this introduction sufficient material has been presented to convince the reader that there is a complete lack of consistency in all the data concerning high explosives. This lack of consistency being due in part to the fact that no theoretical basis for experimentation has been developed. It is proposed therefore to put forth a theory with the hope that it may result in the determination of facts which in turn will disprove it, and make possible the construction of a new and more workable theory or complete it and make explosives a definite science. The effort put forth, therefore, is entirely in the spirit expressed by Guglielmo Ferrero [29] who is here

[27] "Chemiker-Zeitung," 1925, page 622.

[28] "Zeit. fur d. Gesamte S. u S.," 1925, Nr. II, 12 and 21; 1926, Nr. 1–9.

[29] "Les Lois Psychologiques du Symbolisme," 1895, Preface, page 8.

quoted, "Therefore theory which gives to facts their value and significance is often very useful even if it is partially false, for it throws light on phenomena which no one observed, it forces an examination, from many angles, of facts which no one had hitherto studied, and it gives the impulse for more extended and more productive researches."

RECAPITULATION.

1. A new phenomenon was discovered with the property of mercury fulminate to initiate detonation.

2. Theories to explain the nature of this phenomenon have been advanced.

3. The principal theory depends upon the idea of a *preliminary heating* or ignition temperature.

4. A relation between ignition temperature and sensitiveness cannot be established.

5. Every means of determining sensitiveness is at variance with every other means.

6. The *sine qua non* of an explosive is sensitiveness. Some of the most violent explosives were used commercially for many years and considered safe because the force to which they were sensitive had not been discovered.

7. Formulæ have been written to classify explosives. These have all been based on assumptions.

8. No real progress has been made in the knowledge of high explosives since their discovery.

It would seem that the investigation of explosives should be attempted from an entirely new angle.

In a subsequent part of this paper the writer will suggest a new theory and attempt to demonstrate the direction that theoretical investigation of high explosives should proceed.

The rates of detonation used in the development of the theory were determined by the Bureau of Mines of the Department of the Interior and are published with the permission of that organization.

OPERATIONAL METHODS IN THE THEORY OF AIRFOILS IN NON-UNIFORM MOTION.

BY

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In this paper Heaviside's operators are employed to obtain the following results:

1. New integral expressions convenient for numerical calculation are obtained for the lift and circulation of a two-dimensional thin airfoil following a sudden change of its angle of attack or velocity.

2. Approximate expressions are obtained for the initial and final behaviors of these functions.

3. The circulation function is shown to be identical with the function that expresses the lift of an airfoil during and following its entrance into a sharp-edged vertical gust.

It appears that in problems concerning the non-uniform motion of two-dimensional thin airfoils the mathematical analysis is most conveniently carried out by the use of Heaviside's operators.

1. INTRODUCTION: THE THEORY OF THIN AIRFOILS IN TWO-DIMENSIONAL VARIABLE MOTION.

This theory has recently been presented in a new form by Th. von Kármán and the present author.[1] The airfoil was replaced by a plane [2] sheet of bound vortices, and, in accordance with the condition that the total circulation in the fluid remain equal to zero, variations of the circulation about the airfoil were assumed to produce a "wake" of spanwise vortices behind the airfoil. This wake was also assumed to consist of a plane vortex sheet. The strength and distribution of the circulation of the bound vortex sheet were determined by the

[1] "Airfoil Theory for Non-Uniform Motion," *Journal of the Aeronautical Sciences*, **5** (1938), pp. 379–390.

[2] The effects of curvature and thickness of the airfoil are, to a first approximation, simply additive constants.

condition of vanishing normal velocity together with the condition of zero or finite velocity at the trailing edge at every instant.

On the basis of these approximations the system was considered as made up of vortex pairs, and its Kelvin impulse [3] (per unit span) was calculated. The lift force on the airfoil was then obtained as the time rate of change of the impulse. One of the principal simplifications obtained in the paper was the expression of the instantaneous lift in three parts:

$$L(t) = L_0(t) + L_1(t) + L_2(t), \tag{1}$$

where

$$L_0(t) = \rho U(t)\Gamma_0(t), \tag{2}$$

$$L_1(t) = -\rho \frac{d}{dt}\int_{-c}^{c} \gamma_0(x, t)x\,dx, \tag{3}$$

$$L_2(t) = \rho U(t)c\int_{c}^{R(t)} \gamma(\xi, t)d\xi/\sqrt{\xi^2 - c^2}. \tag{4}$$

In these expressions a coordinate system fixed relative to the airfoil has been chosen, $U(t)$ being the velocity of the undisturbed fluid. The bound and wake vortex sheets are assumed to lie in the direction of $U(t)$, and the downstream distance from the centerline of the bound vortex sheet is called x for points on the bound sheet and ξ for points on the wake sheet. The chord of the airfoil is taken equal to $2c$, so that $x = -c$ represents the upstream edge of the bound vortex sheet, and $x = \xi = c$ represents the downstream edge, which is the upstream edge of the wake. The line $\xi = R(t)$ represents the downstream extremity of the wake. The density of the fluid is ρ, and t is the time variable. The function $\gamma(\xi, t)$ is the strength of the wake vortex sheet at ξ; the function $\gamma_0(x, t)$ is the bound vortex strength at x that would be calculated if the usual formulae [4] of the stationary thin airfoil theory were applied and the velocities induced by the wake vortices were neglected. Hence $\gamma_0(x, t)$ depends only on the instantaneous relative motion of the airfoil and the undisturbed fluid, and is calculated without knowledge of the wake vortex strength $\gamma(\xi, t)$. Since $\Gamma_0(t) = \int_{-c}^{c} \gamma_0(x, t)dx$, it also is

[3] See, e.g., H. Lamb, "Hydrodynamics," 6th ed., p. 161, p. 214.

[4] See, e.g., Durand, "Aerodynamic Theory," vol. II, chapt. II.

dependent only upon the instantaneous relative motion of the airfoil and the undisturbed fluid and is calculated by use of steady-motion formulae.[4] $L_1(t)$ is also obtainable as soon as $\gamma_0(x, t)$ is known, and therefore the principal problem in any application of these results is the calculation of $L_2(t)$, which gives the wake effect.

The determination of $L_2(t)$ implies the knowledge of the wake function $\gamma(\xi, t)$. In the v. Kármán-Sears paper it is shown that this function is related to the motion of the wing by the relation

$$\Gamma_0(t) = -\int_c^{R(t)} \gamma(\xi, t) \sqrt{\frac{\xi + c}{\xi - c}} d\xi. \tag{5}$$

This is an integral equation to be solved for $\gamma(\xi, t)$ when $\Gamma_0(t)$ is specified.

The total circulation about the airfoil, $\Gamma_T(t)$, may be calculated if $\gamma(\xi, t)$ is determined, since the total circulation in the fluid is

$$\Gamma_T(t) + \int_c^{R(t)} \gamma(\xi, t) d\xi = 0 \tag{6}$$

as pointed out above.

It is assumed by v. Kármán and Sears that the wake vortices are carried away from the airfoil with the free-stream velocity $U(t)$; i.e., that they are stationary relative to the undisturbed fluid. The function $\gamma(\xi, t)$ is then a function of the single variable $\xi - \sigma$, where $\sigma = \int_0^t U(t)dt$. If $t = 0$ represents the beginning of the motion, then $\sigma = R(t) - c$, and $\xi - \sigma$ is the distance from the downstream end of the wake to the point ξ in the wake. Now $\gamma(\xi, t)$ may be written as a function $\mu(s)$ where $s = c + \sigma - \xi = R(t) - \xi$, and (4), (5), and (6) may be written

$$L_2(t) = \rho U(t) c \int_0^\sigma \mu(s) ds / \sqrt{2(\sigma - s) + (\sigma - s)^2}, \tag{7}$$

$$\Gamma_0(t) = -\int_0^\sigma \mu(s) \sqrt{\frac{2c + \sigma - s}{\sigma - s}} ds, \tag{8}$$

$$\Gamma_T(t) = -\int_0^\sigma \mu(s) ds. \tag{9}$$

2. THE PROBLEM OF A SUDDEN CHANGE OF $\Gamma_0(t)$ (WAGNER'S PROBLEM).

H. Wagner [5] first considered the case in which the velocity and angle of attack of the airfoil are constant up to a certain instant at which the velocity and/or angle of attack change suddenly and remain constant thereafter. If the instant of change is taken as $t = 0$, then $\Gamma_0(t)$ in this case is equal to a constant plus a term proportional to Heaviside's "unit step function" $1(t)$, which is zero for $t < 0$ and unity for $t \geqq 0$. Since $U(t)$ is constant for $t \geqq 0$, $\sigma = Ut$ for $\sigma \geqq 0$, and, neglecting the constant terms, we may write for (7), (8), and (9),

$$L_2(t) = -\rho U \Phi(\sigma), \qquad \text{where}$$

$$\Phi(\sigma) = -c \int_0^\sigma \mu(s) ds / \sqrt{2(\sigma - s) + (\sigma - s)^2}, \quad (10)$$

$$\Gamma_0(t) = 1(\sigma) = -\int_0^\sigma \mu(s) \sqrt{\frac{2c + \sigma - s}{\sigma - s}} ds, \quad (11)$$

$$\Gamma_T(t) = \Psi(\sigma) \qquad \text{where} \qquad \Psi(\sigma) = -\int_0^\sigma \mu(s) ds. \quad (12)$$

For $t \geqq 0$, $\gamma_0(x, t)$ is constant in this case; hence, from (3), $L_1(t) = 0$. Also $L_0(t) = \rho U$ for $t \geqq 0$, and therefore the total lift is simply

$$L(t) = \rho U \{1 - \Phi(\sigma)\}. \quad (13)$$

The function $\{1 - \Phi(\sigma)\}$ has been called Wagner's function.

In the following analysis, it will be assumed that $c = 1$; i.e., that all lengths are measured in terms of the half chord. Equation (11) is then the same as the integral equation employed by Wagner.[6] A convenient method of solution of (10) to (12) is found by expressing them in terms of Heaviside's operators.[7]

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[5] "Dynamischer Auftrieb von Tragflügeln," *Zeitschr. f. angew. Math. u. Mech.*, Bd. 5 (1925), pp. 17–35.

[6] Loc. cit.

[7] See, e.g., Jeffreys, "Operational Methods in Mathematical Physics," (Cambridge Univ. Tract), 2nd ed.

3. THE OPERATIONAL FORMS.

If $\bar{f}(p)$ is the Heaviside operator whose interpretation is $f(\sigma)$, the operational form of $f(\sigma)$ is given by Carson's integral [8]

$$\bar{f}(p) = p\int_0^\infty e^{-p\sigma} f(\sigma)d\sigma \tag{14}$$

and the interpretation of the operational form $\bar{f}(p)$ is given by Bromwich's integral [9]

$$f(\sigma) = \frac{1}{2\pi i}\int_N e^{p\sigma}\bar{f}(p)dp/p, \tag{15}$$

where the contour N is drawn from $\alpha - i\infty$ to $\alpha + i\infty$, where α is positive and finite, such that all singularities of $\bar{f}(p)$ lie to the left of the contour. Carson's integral gives the Laplace transform of the function,[10] and Bromwich's integral gives its solution by means of the Mellin inversion formula.[11] The operational notation shortens the writing and allows the use of the known results of the operational calculus.

To solve (10) to (12) we use the "product rule" [12]

$$\frac{1}{p}\bar{f}_1(p)\bar{f}_2(p) = \int_0^\sigma f_1(s)f_2(\sigma - s)ds \tag{16}$$

where the equality sign is used to denote that the right side is the interpretation of the operator on the left. Then (10) to (12) become, with $c = 1$,

$$\bar{\Phi}(p) = -\frac{1}{p}\bar{\mu}(p)\bar{g}(p), \tag{17}$$

$$1 = -\frac{1}{p}\bar{\mu}(p)\bar{f}(p), \tag{18}$$

$$\bar{\Psi}(p) = -\frac{1}{p}\bar{\mu}(p), \tag{19}$$

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[8] Jeffreys, loc. cit., p. 33. The notation of the present paper differs from that of Jeffreys in that σ is used in place of t as the independent variable.

[9] Ibid., p. 22.

[10] See Bateman, "Report of the British Association" (Sheffield, 1910), p. 393; and *Messenger of Math.*, **49** (1920), pp. 134–137.

[11] See Hardy, *Messenger of Math.*, **50** (1921), pp. 165–171.

[12] Van der Pol, "Symbolic Calculus," *Phil. Mag.*, **13** (1932), p. 541.

where $g(\sigma) = 1/\sqrt{2\sigma + \sigma^2}$ and $f(\sigma) = \sqrt{(2 + \sigma)/\sigma}$, so that, from (14),[13]

$$\bar{g}(p) = p\int_0^\infty e^{-p\sigma}d\sigma/\sqrt{2\sigma + \sigma^2} = pe^pK_0(p), \qquad (20)$$

and, similarly,[14]

$$\bar{f}(p) = p\int_0^\infty e^{-p\sigma}\sqrt{\frac{2 + \sigma}{\sigma}}d\sigma = pe^p\{K_0(p) + K_1(p)\}, \quad (21)$$

where $K_0(p)$ and $K_1(p)$ are modified Bessel functions of the second kind. Hence, dividing (17) and (19) by (18),

$$\bar{\Phi}(p) = \bar{g}(p)/\bar{f}(p) = K_0(p)\{K_0(p) + K_1(p)\}^{-1}, \qquad (22)$$

$$\bar{\Psi}(p) = 1/\bar{f}(p) = p^{-1}e^{-p}\{K_0(p) + K_1(p)\}^{-1}. \qquad (23)$$

The functions $\Phi(\sigma)$ and $\Psi(\sigma)$ may now be evaluated by application of Bromwich's integral, (15).[15] H. G. Küssner [16] has calculated $\Phi(\sigma)$ from such an integral, although he obtained it by a different—and more complicated—analysis. More convenient methods are given below. We first notice an immediate interesting application of the operators given above.

4. APPLICATION TO THE CASE OF STEADY OSCILLATIONS.

Expressions for the lift and circulation in cases of steady-state harmonic oscillation (i.e., $\Gamma_0(t) = Ae^{i\omega Ut} = Ae^{i\omega\sigma}$, where A and ω are real constants) can be written immediately, since the operator that gives the response of a linear system to $1(\sigma)$ is equal to the steady-state response to $e^{i\omega\sigma}$ if p is replaced by $i\omega$ and the factor $e^{i\omega\sigma}$ is applied. Hence the lift in this case is

$$\begin{aligned} L(t) &= L_0(t) + L_1(t) + L_2(t) \\ &= L_0(t) + L_1(t) - \rho UAe^{i\omega\sigma}\bar{\Phi}(i\omega), \end{aligned}$$

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[13] Gray, Mathews, MacRobert, "Bessel Functions," p. 50, eq. following (29).

[14] Ibid: The equivalence of the forms is seen by putting $\sigma + 1 = \xi$ and using eq. (29) of this reference.

[15] The analysis of the Appendix proves that $\bar{\Phi}(p)$ and $\bar{\Psi}(p)$ have no singularities to the right of the contour N. Consequently, integrals of the type (15) do indeed give solutions to the integral equations; in fact the constant α may be taken as small as desired.

[16] "Zusammenfassender Bericht über den instationären Auftrieb von Flügeln," *Luftfahrtforschung*, Bd. 13 (1936), p. 421.

i.e., since $L_0(t) = \rho U \Gamma_0(t) = \rho U A e^{i\omega\sigma}$ in this case,

$$L(t) = L_1(t) + L_0(t)\{1 - \bar{\Phi}(i\omega)\}. \tag{24}$$

The circulation may be expressed analogously in terms of $\bar{\Psi}(i\omega)$. The function

$$\{1 - \bar{\Phi}(i\omega)\} = K_1(i\omega)\{K_0(i\omega) + K_1(i\omega)\}^{-1}$$

is called Theodorsen's function,[17] and is important in the theory of wing flutter. It is seen (cf. (13)) that Theodorsen's function is obtained by replacing p by $i\omega$ in the operational form of Wagner's function.[18]

5. INTERPRETATION OF THE OPERATORS.

In the present cases it can be shown that the integral along the contour N in (15) may be replaced by the integral along M in Fig. 1.[19] To prove this, we consider the integral of (15) with $\bar{f}(p)$ replaced by $\bar{\Phi}(p)$ and $\bar{\Psi}(p)$, respectively, from (22) and (23), and note (*i*) that in both cases the integrals along the quadrants Q_1 and Q_2 of the large circle vanish as $P \to \infty$,[20] and (*ii*) that the integrands have no poles inside the entire contour formed by N, Q_1, M, and Q_2.[21]

Therefore, considering first $\Phi(\sigma)$, we have

$$\begin{aligned}\Phi(\sigma) &= \frac{1}{2\pi i}\int_N e^{p\sigma}\bar{\Phi}(p)dp/p \\ &= \frac{1}{2\pi i}\lim_{\epsilon\to 0}\left\{\int_\infty^\epsilon e^{-x\sigma}\bar{\Phi}(xe^{-\pi i})dx/x \right. \\ &\qquad \left. + \int_\epsilon^\infty e^{-x\sigma}\bar{\Phi}(xe^{\pi i})dx/x + \int_{-\pi}^{\pi}\bar{\Phi}(\epsilon e^{i\theta})id\theta\right\}. \end{aligned} \tag{25}$$

[17] Cf. v. Kármán-Sears, loc. cit., eqs. (29), (30), (33); also Th. Theodorsen, "General Theory of Aerodynamic Instability and the Mechanism of Flutter," N.A.C.A. Tech. Rep. No. 496 (1935); also Küssner, loc. cit., p. 415.

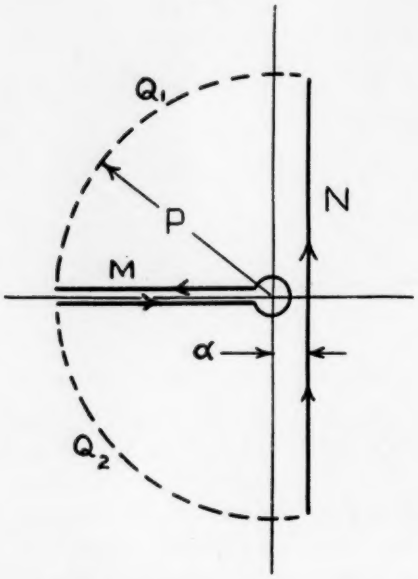
[18] This is the "reciprocity" which was pointed out by I. Garrick: "On Some Reciprocal Relations in the Theory of Non-Stationary Flows," N.A.C.A. Tech. Rep. No. 627 (1939).

[19] This method of solution of Laplace's integral equation was suggested by Bateman: *Proc. Camb. Phil. Soc.*, **15** (1910), p. 427.

[20] The proof of this statement is analogous to the proof of Jordan's lemma.

[21] The proof of this statement is given in the Appendix to this paper.

FIG. 1.



The Bessel functions along the negative real axis are [22]

$$\left.\begin{aligned} K_0(xe^{-\pi i}) &= K_0(x) + \pi i I_0(x); \\ K_0(xe^{\pi i}) &= K_0(x) - \pi i I_0(x); \\ K_1(xe^{-\pi i}) &= -K_1(x) + \pi i I_1(x); \\ K_1(xe^{\pi i}) &= -K_1(x) - \pi i I_1(x), \end{aligned}\right\} \tag{26}$$

where $I_0(x)$ and $I_1(x)$ are modified Bessel functions of the first kind. Using these forms in (25), with $\bar{\Phi}(p)$ from (22), we have [23]

$$\begin{aligned} \Phi(\sigma) &= -\frac{1}{2\pi i}\int_0^\infty \left\{ \frac{K_0 + \pi i I_0}{K_0 - K_1 + \pi i(I_0 + I_1)} \right. \\ &\qquad \left. - \frac{K_0 - \pi i I_0}{K_0 - K_1 - \pi i(I_0 + I_1)} \right\} e^{-x\sigma}\frac{dx}{x} \\ &= \int_0^\infty \left\{ \frac{K_0 I_1 + I_0 K_1}{(K_0 - K_1)^2 + \pi^2(I_0 + I_1)^2} \right\} e^{-x\sigma}\frac{dx}{x} \\ &= \int_0^\infty \{(K_0 - K_1)^2 + \pi^2(I_0 + I_1)^2\}^{-1} e^{-x\sigma} dx/x^2, \end{aligned} \tag{27}$$

[22] Cf. Watson: *Bessel Functions*, p. 80.

[23] The last integral in (25) vanishes as $\epsilon \to 0$.

where (x) is implied as the argument of the Bessel functions and use has been made of the identity $K_0I_1 + I_0K_1 = 1/x$.[24] The analogous expression for $\Psi(\sigma)$ is obtained by a similar analysis; it is

$$\Psi(\sigma) = 1 - \int_0^\infty \{(K_0 - K_1)^2 + \pi^2(I_0 + I_1)^2\}^{-1} \times (I_0 + I_1)e^{-x(\sigma-1)}dx/x^2. \quad (28)$$

The integrals in (27) and (28) are both rapidly convergent, and can easily be evaluated by numerical methods.

6. ASYMPTOTIC SOLUTIONS FOR LARGE POSITIVE VALUES OF σ.

Since the contour N in (15) may be replaced by the contour M, asymptotic expansions for $\Phi(\sigma)$ and $\Psi(\sigma)$ may be obtained by expanding $\bar{\Phi}(p)$ and $\bar{\Psi}(p)$ in expansions that would be suitable for small values of p, i.e., in ascending powers, and then interpreting the operators term by term according to the known rules of the Heaviside calculus.[25]

Substituting the ascending power series for the Bessel functions in (22) and (23), we have

$$\bar{\Phi}(p) = \frac{-\left\{1 + \frac{p^2}{4} + \cdots\right\}\left(\log\frac{p}{2} + \gamma\right) + \frac{p^2}{4} + \cdots}{\frac{1}{p}\left\{1 - p\left(\log\frac{p}{2} + \gamma\right) + \frac{p^2}{2}\log\frac{p}{2} - \frac{p^3}{4}\left(\log\frac{p}{2} + \gamma - 1\right) + \cdots\right\}}$$

$$= -p\left(\log\frac{p}{2} + \gamma\right) - p^2\left(\log\frac{p}{2} + \gamma\right)^2 - p^3\left\{\frac{1}{4}\left(\log\frac{p}{2} + \gamma - 1\right) - \frac{1}{2}\log\frac{p}{2}\left(\log\frac{p}{2} + \gamma\right) + \left(\log\frac{p}{2} + \gamma\right)^3\right\} + O(p^4[\log p]^4), \quad (29)$$

[24] Gray, Mathews, MacRobert, loc. cit., p. 25, eq. (53).

[25] Cf. S. Goldstein, "The Application of Heaviside's Operational Method to the Solution of a Problem in Heat Conduction," *Zeitschr. f. angew. Math. u. Mech.*, Bd. 12 (1932), p. 234.

$$\overline{\Psi}(p) = e^{-p}\left\{1 + p\left(\log\frac{p}{2} + \gamma\right) + p^2\left[\left(\log\frac{p}{2} + \gamma\right)^2 - \frac{1}{2}\log\frac{p}{2}\right] + p^3\left[\left(\log\frac{p}{2} + \gamma\right)^3 - \log\frac{p}{2}\left(\log\frac{p}{2} + \gamma\right) + \frac{1}{4}\left(\log\frac{p}{2} + \gamma - 1\right)\right] + O(p^4[\log p]^4)\right\}, \quad (30)$$

where $\gamma = 0.5772\cdots$ is Euler's constant.

The operators of the form $p^m\left[\log\frac{p}{2}\right]^n$ can be interpreted by a method used by Goldstein.[26] When we have deformed the contour N into M, the following equation holds for all values of n:

$$\frac{1}{p^n} = \frac{\sigma^n}{\Gamma(n+1)}. \quad (31)$$

This may be written in the form

$$\left(\frac{p}{2}\right)^s = \frac{(2\sigma)^{-s}}{\Gamma(1-s)}$$

and differentiation with respect to s gives

$$\left(\frac{p}{2}\right)^s \log(p/2) = -\frac{1}{\pi}(2\sigma)^{-s}\log(2\sigma)\Gamma(s)\sin(s\pi) + \frac{1}{\pi}(2\sigma)^{-s}\Gamma'(s)\sin(s\pi) + (2\sigma)^{-s}\Gamma(s)\cos(s\pi), \quad (32)$$

i.e., for integral values of s,

$$p^s \log(p/2) = (-)^s\Gamma(s)\sigma^{-s}. \quad (33)$$

By repeated differentiation of (32), the following formulæ are obtained for integral values of s:

$$p^s[\log(p/2)]^2 = 2(-)^s\{\Gamma'(s) - \Gamma(s)\log(2\sigma)\}\sigma^{-s}, \quad (34)$$

$$p^s[\log(p/2)]^3 = (-)^s\{3\Gamma(s)[\log(2\sigma)]^2 - 6\Gamma'(s)\log(2\sigma) + 3\Gamma''(s) - \pi^2\Gamma(s)\}\sigma^{-s}. \quad (35)$$

[26] "Operational Representations of Whittaker's Confluent Hypergeometric Function and Weber's Parabolic Cylinder Function," *Proc. Lond. Math. Soc.*, Ser. 2, vol. 34 (1931), p. 111.

On the contour M operators of the form p^m, where m is a positive integer, do not contribute to the result (as we can see from (31)). Therefore, the operator in (29) can be evaluated by means of (33) to (35). We obtain

$$\Phi(\sigma) \sim \frac{1}{\sigma} + \frac{2\log(2\sigma)}{\sigma^2} - \frac{2}{\sigma^2} + \frac{6[\log(2\sigma)]^2}{\sigma^3} - \frac{16\log(2\sigma)}{\sigma^3} + (7/2 + \gamma - \pi^2)/\sigma^3 + \cdots. \quad (36)$$

The operator in (30) is interpreted by means of (33) to (35) and the rule [27]

$$e^{-\lambda p}\bar{f}(p) = f(\sigma - \lambda) \text{ for } \sigma > \lambda. \quad (37)$$

We obtain

$$\Psi(\sigma) \sim 1 - \frac{1}{\sigma - 1} - \frac{2\log(2\sigma - 2)}{(\sigma - 1)^2} + \frac{3/2}{(\sigma - 1)^2} - \frac{6[\log(2\sigma - 2)]^2}{(\sigma - 1)^3} + \frac{14\log(2\sigma - 2)}{(\sigma - 1)^3} - \frac{1/2 + 2\gamma - \pi^2}{(\sigma - 1)^3} + \cdots. \quad (38)$$

7. SOLUTIONS FOR SMALL VALUES OF σ.

To evaluate $\Phi(\sigma)$ and $\Psi(\sigma)$ for small values of σ, we replace the Bessel functions in (22) and (23) by their asymptotic expansions, expand in descending powers of p, and interpret term by term.[28] We then have [29]

$$\bar{\Phi}(p) \sim \frac{\sqrt{\frac{\pi}{2p}}\, e^{-p}\left\{1 - \frac{1}{8p} + \frac{9}{128p^2} - \frac{75}{1024p^3} + \cdots\right\}}{\sqrt{\frac{\pi}{2p}}\, e^{-p}\left\{2 + \frac{1}{4p} - \frac{3}{64p^2} + \frac{15}{512p^3} - \cdots\right\}} \sim \frac{1}{2}\left\{1 - \frac{1}{4p} + \frac{1}{8p^2} - \frac{7}{64p^3} + \cdots\right\}, \quad (39)$$

[27] Van der Pol, loc. cit., p. 540.

[28] Cf. Goldstein, *Zeitschr. f. angew. Math. u. Mech.*, loc. cit.

[29] The asymptotic expression for $K_n(z)$ is given in Gray, Mathews, MacRobert, loc. cit., p. 55 and p. xiii.

$$\bar{\Psi}(p) \sim \sqrt{\frac{2}{\pi p}}\left\{2 + \frac{1}{4p} - \frac{3}{64p^2} + \frac{15}{512p^3} - \cdots\right\}^{-1}$$
$$\sim \frac{1}{\sqrt{2\pi p}}\left\{1 - \frac{1}{8p} + \frac{5}{128p^2} - \frac{23}{1024p^3} + \cdots\right\}. \qquad (40)$$

Interpreting according to (31), we have

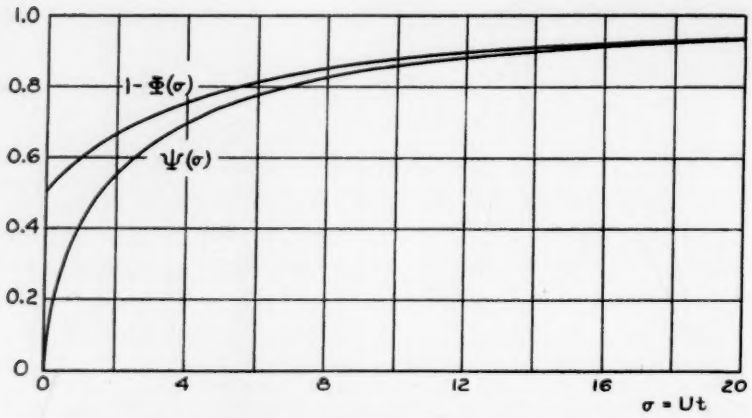
$$\Phi(\sigma) = \frac{1}{2} - \frac{\sigma}{8} + \frac{\sigma^2}{32} - \frac{7}{768}\sigma^3 + \cdots, \qquad (41)$$

$$\Psi(\sigma) = \frac{\sqrt{2\sigma}}{\pi}\left\{1 - \frac{\sigma}{12} + \frac{\sigma^2}{96} - \frac{23}{13440}\sigma^3 + \cdots\right\}. \qquad (42)$$

8. NUMERICAL RESULTS.

Graphs of the functions $\{1 - \Phi(\sigma)\}$ (Wagner's function) and $\Psi(\sigma)$ are shown in Fig. 2. The numerical data are given

FIG. 2.



in Table 1, in which the ranges of applicability of the various approximate expressions are indicated.

9. APPLICATION TO THE CASE OF A SHARP-EDGED GUST.

The numerical results obtained here are in good agreement with those of Wagner for $\sigma \leqq 10$. Wagner's result for $\{1 - \Phi(\sigma)\}$ has been applied by v. Kármán and Sears [30] to

[30] Loc. cit.

TABLE I.

σ.	$1-\Phi(\sigma)$.			$\Psi(\sigma)$.		
	Eq. (27).	Eq. (41).	Eq. (36).	Eq. (28).	Eq. (42).	Eq. (38).
0	1/2	1/2	—	0	0	—
1/4	0.529	0.529	—	0.221	0.221	—
1/2	0.556	0.556	—	0.306	0.306	—
3/4	0.580	0.580	—	0.368	0.368	—
1	0.601	0.603	—	0.417	0.417	—
3/2	0.637	0.648	—	0.496	0.492	—
2	0.669	—	—	0.552	0.548	—
4	0.758	—	—	0.697	—	—
6	0.813	—	—	0.774	—	—
8	0.849	—	—	0.823	—	—
10	0.875	—	0.860	0.856	—	—
15	0.915	—	0.909	0.905	—	0.898
20	0.937	—	0.935	0.931	—	0.928
25	0.949	—	0.949	0.945	—	0.945
30	0.959	—	0.959	0.956	—	0.956
40	0.970	—	0.970	0.969	—	0.969

the calculation of the growth of the lift of a rigid thin airfoil during and following its penetration into the boundary of a sharp-edged vertical gust. The operational treatment of this important problem yields certain simplifications.

Equations (1) to (4) are again applied for the determination of $L(t)$. We assume that the vertical gust velocity is of unit magnitude and that the velocity $U(t)$ is constant, and the airfoil chord is again taken equal to 2. By application of the results of the stationary airfoil theory [31] for this case, it is found that

$$\left.\begin{aligned} \Gamma_0(t) &= 2\{\cos^{-1}(1-\sigma) - \sqrt{2\sigma-\sigma^2}\} \quad \text{for } 0 \leqq \sigma \leqq 2 \\ &= 2\pi \quad \text{for} \quad \sigma \geqq 2 \\ &= F(\sigma), \quad \text{say;} \end{aligned}\right\} \tag{43}$$

$$\begin{aligned} L_0(t) + L_1(t) &= 2\rho U \cos^{-1}(1-\sigma) \quad \text{for} \quad 0 \leqq \sigma \leqq 2 \\ &= 2\pi\rho U \quad \text{for} \quad \sigma \geqq 2 \\ &= \Lambda_1(\sigma), \quad \text{say;} \end{aligned} \tag{44}$$

where again $\sigma = Ut$, and $t = 0$ is the instant the leading edge begins to penetrate the gust boundary. In the interval $0 \leqq \sigma \leqq 2$, the airfoil passes through the boundary; for $\sigma > 2$ it is entirely within the gust.

[31] See v. Kármán-Sears, loc. cit., p. 388; also Durand, loc. cit.

Since $-\rho U\Phi(\sigma)$ is the value of $L_2(t)$ in the case of a unit "step" of $\Gamma_0(t)$, it can be used to calculate $L_2(t)$ in the present case if $\Gamma_0(t) = F(\sigma)$ is imagined to be made up of infinitesimal steps, $F'(\sigma)d\sigma$. The term $L_2(t)$ is then the result of superposition of the responses to these steps, and is equal to [32]

$$L_2(t) = -\rho U\int_0^\sigma F'(s)\Phi(\sigma - s)ds = \Lambda_2(\sigma), \quad \text{say.} \tag{45}$$

In operational form, using (16),

$$\bar{\Lambda}_2(p) = -\rho U\frac{1}{p}\bar{F}'(p)\bar{\Phi}(p) \tag{46}$$

and, from (14) and (43), the operational form of $F'(\sigma)$ is

$$\begin{aligned}\bar{F}'(p) &= 2p\int_0^2 e^{-p\sigma}\sqrt{\sigma/(2-\sigma)}d\sigma \\ &= 2\pi pe^{-p}\{I_0(p) - I_1(p)\}.^{33}\end{aligned} \tag{47}$$

Similarly the operational form of (44) is

$$\begin{aligned}\bar{\Lambda}_1(p) &= 2\rho Up\int_0^2 e^{-p\sigma}\cos^{-1}(1-\sigma)d\sigma \\ &\quad + 2\pi\rho Up\int_2^\infty e^{-p\sigma}d\sigma = 2\pi\rho Ue^{-p}I_0(p).^{34}\end{aligned} \tag{48}$$

Hence, if we write the total lift $L(t) = L_0(t) + L_1(t) + L_2(t)$ as $\Lambda(\sigma) = \Lambda_1(\sigma) + \Lambda_2(\sigma)$, we have, from (46) to (48),

$$\bar{\Lambda}(p) = 2\pi\rho Ue^{-p}\{I_0(p) - [I_0(p) - I_1(p)]\bar{\Phi}(p)\} \tag{49}$$

and taking $\bar{\Phi}(p)$ from (22) this is

$$\begin{aligned}\bar{\Lambda}(p) &= 2\pi\rho Ue^{-p}\left\{I_0(p) - \frac{K_0(p)[I_0(p) - I_1(p)]}{K_0(p) + K_1(p)}\right\} \\ &= 2\pi\rho Ue^{-p}\left\{\frac{K_0(p)I_1(p) + K_1(p)I_0(p)}{K_0(p) + K_1(p)}\right\} \\ &= 2\pi\rho Ue^{-p}p^{-1}\{K_0(p) + K_1(p)\}^{-1},\end{aligned} \tag{50}$$

[32] This integral is sometimes called "Duhamel's superposition integral."

[33] Cf. Gray, Mathews, MacRobert, loc. cit., p. 46, eq. (11), letting $\sigma = 1 - \xi$ and integrating one term by parts.

[34] Ibid., p. 46, integrating once by parts and letting $\sigma = 1 - \xi$.

since $K_0(p)I_1(p) + K_1(p)I_0(p) = p^{-1}$. Comparing (50) and (23) we see that

$$\bar{\Lambda}(p) = 2\pi\rho U\bar{\Psi}(p), \tag{51}$$

i.e.,

$$L(t) = \Lambda(\sigma) = 2\pi\rho U\Psi(\sigma). \tag{52}$$

Hence the function $L(t)/2\pi\rho U$ in the sharp-gust case is identically the same as $\Gamma_T(t)$ in Wagner's case as given by (28), (38), and (42). This rather surprising result may be verified by comparing the numerical results given here for $\Gamma_T(t)$ with those given by v. Kármán and Sears for the lift in the case of the sharp gust.

10. CONCLUSION.

The functions calculated herein may be employed to calculate the lift and circulation of thin airfoils in arbitrary two-dimensional motions. Such applications arise, for example, in problems of airplane dynamics. The use of Heaviside's operational methods in such problems has been suggested by R. T. Jones.[35]

APPENDIX: SINGULARITIES OF THE OPERATORS.

The excess of the number of zeros over the number of poles of any analytic function $\varphi(z)$ within the contour C is equal to $\frac{1}{2\pi i}\int_C \frac{\varphi'(z)}{\varphi(z)}\,dz$.[36] To show that the integrands considered in section 5 have no poles within the contour formed by N, Q_1, M, and Q_2 of Fig. 1, it will be sufficient to show that this integral vanishes when $\varphi(z)$ is taken equal to $\bar{\Phi}(z)$ in (22) and C is taken as the contour of Fig. 3. This suffices because the other factors involved in the two integrands have no poles or zeros and $\bar{\Phi}(z)$ has no zeros [37] within this contour. Hence we consider the integral

$$\int_C \{\bar{\Phi}'(z)/\bar{\Phi}(z)\}dz = \int_C \left\{ \frac{K_0^2 + K_0K_1/z - K_1^2}{K_0(K_0 + K_1)} \right\} dz, \tag{53}$$

[35] "Operational Treatment of the Non-Uniform Lift Theory in Airplane Dynamics," N.A.C.A. Tech. Note 667 (1938).

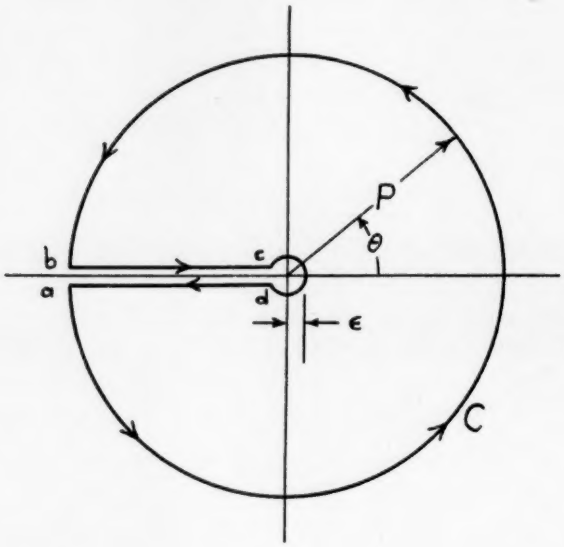
[36] Whittaker and Watson, "Modern Analysis," 4th ed., p. 119.

[37] See, e.g., Watson's "Bessel Functions," Chapters III and XV.

where $K_0 = K_0(z)$ and $K_1 = K_1(z)$. We consider the contour in three parts:

(i) From a to b, $z = Pe^{i\theta}$; for large P, the asymptotic expansions [38] for $K_n(z)$, valid for $-3\pi/2 < \arg z < 3\pi/2$, may

FIG. 3.



be used; then

$$\bar{\Phi}'(z)/\bar{\Phi}(z) \sim \frac{1 - \frac{1}{4z} + \frac{1}{z} - \left(1 + \frac{3}{4z}\right) + O(z^{-2})}{\left(1 - \frac{1}{8z}\right)\left(2 + \frac{1}{4z}\right) + \cdots} = O(z^{-2}).$$

Hence this part of the integral vanishes as $P \to \infty$.

(ii) The integrals from b to c and from d to a reduce to

$$\log \left\{ \bar{\Phi}(xe^{-\pi i})/\bar{\Phi}(xe^{\pi i}) \right\} \Big|_{x=\epsilon}^{x=P}$$

and, substituting for the Bessel functions according to (26),

[38] Gray, Mathews, MacRobert, loc. cit., p. 55 and p. xiii.

we find this to be equal to

$$\log\left\{\frac{K_0+\pi iI_0}{K_0-\pi iI_0}\cdot\frac{K_0-K_1-\pi i(I_0+I_1)}{K_0-K_1+\pi i(I_0+I_1)}\right\}\Bigg|_{\epsilon}^{P}$$
$$=2i\left\{\tan^{-1}\left(\frac{\pi I_0}{K_0}\right)-\tan^{-1}\left(\frac{\pi(I_0+I_1)}{K_0-K_1}\right)\right\}\Bigg|_{\epsilon}^{P},$$

where (x) is implied as the argument of the Bessel functions. The limit of this quantity as $P\to\infty$ and $\epsilon\to 0$ is easily shown to be $2\pi i$.

(iii) The integral from c to d is

$$\int_{\pi}^{-\pi}\{\bar{\Phi}'(\epsilon e^{i\theta})/\bar{\Phi}(\epsilon e^{i\theta})\}\epsilon e^{i\theta}id\theta,$$

whose limit as $\epsilon\to 0$ is

$$\lim_{\epsilon\to 0}\int_{\pi}^{-\pi}\frac{K_0K_1/\epsilon e^{i\theta}}{K_0K_1}\epsilon e^{i\theta}id\theta=-2\pi i.$$

Therefore the value of the complete integral is zero, and the proof is complete.

Weather Bureau Speedy in Making Daily Maps.—Speed rules in making up the daily weather maps of the United States Weather Bureau. To gather and digest weather information from more than 350 points in the United States, put it by hand on maps, and translate this information into State and regional forecasts, not a minute can be lost. In the Washington, D. C., map room information on which the daily printed map is based starts coming in shortly after 7.30 o'clock each morning, and finished maps are coming off the press by 11 A.M. From all parts of the United States come barometer readings, temperatures, precipitation, wind direction and velocity, and other weather information. This comes in code and is translated by a reader, who stands at the head of a long table. Seated at both sides of the table are the map makers who note weather conditions in figures and symbols on blank maps. One map gets barometric pressure. Another gets temperatures. Another gets wind direction, and so on. On a master map all this information is noted by a map maker who must have unusual skill and speed. From this master map and such supplementary maps as are needed the forecaster makes his State forecasts—the weather information which appears in the lower, left corner of all maps. As the forecaster dictates his forecasts several copies are typed for messengers and to answer telephone requests, a telegraph operator sends them by wire to all parts of the Washington forecast district, and they also are hand set in type. Because not more than about 200 different words are used ordinarily, the forecasts are set in type a word at a time, rather than letter by letter. The words, and in some cases several words, are cast as one unit of type.

R. H. O.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

CHANGES IN STANDARD FREQUENCY BROADCASTS.

Beginning on May 1, the following changes became effective in the standards of frequency and musical pitch broadcast from the Bureau's radio station WWV at Beltsville, Md.: (*a*) The frequency of 20 megacycles per second has been replaced by 15 mc./sec.; (*b*) announcements are given by voice only, not by telegraphic keying; (*c*) announcements during the musical pitch broadcasting are given every 5 instead of every 10 minutes; (*d*) the ionosphere bulletins have been discontinued.

The standard frequency broadcasts now include: Standard radio frequencies; standard time intervals (seconds pulses); standard audio frequency; and standard musical pitch (440 c./sec.). The last named, and its carrier, a standard 5 mc./sec., are broadcast continuously, day and night.

Those desiring full details should write to the National Bureau of Standards, Washington, D. C., for a copy of Letter Circular LC591, "Standard of Frequency and Musical Pitch Broadcast by National Bureau of Standards."

RADIO SOCIETIES' MEETINGS: IONOSPHERIC CHARACTERISTICS AND RADIO TRANSMISSION.

At the meeting of the American Section, International Scientific Radio Union, held at the National Academy of Sciences on April 26, J. H. Dellinger was elected chairman of the section. At the joint meeting of the section with the Institute of Radio Engineers, Dr. Dellinger and Newbern Smith presented a paper on the prediction of ionospheric characteristics.

The forecasting of monthly average values for ionospheric and radio conditions was undertaken at the Bureau last year. Beginning in the March, 1939, issue of the *Proceedings of the Institute of Radio Engineers*, the Bureau has published each month, in addition to past observed values, predicted values for the month following that of publication. Because of the

* Communicated by the Director.

necessary lag in publication procedure, the predictions had to be made three months in advance.

A set of published predictions as well as observed values is now available for each month throughout a full year. Dr. Dellinger presented and compared these published predictions and observations, and also presented some predictions of longer range for the future.

The monthly data were published in the form of graphs of maximum usable frequencies for dependable radio transmission via the regular layers, giving the monthly average values for undisturbed days, as a function of time of day and for various transmission distances. These graphs also predict critical frequencies and, implicitly, ionization densities of the F and F_2 layers (the number of electrons per cubic centimeter being 0.0124 times the square of the ordinary-wave critical frequency in kilocycles per second).

In announcing the service, the Bureau stated that predicted values were expected to have an accuracy within 15 per cent., and the results show this to have been well met. The limit of accuracy is determined by the minor unpredictable variations of solar activity from its trend in the 11-year cycle.

At the same meeting, T. R. Gilliland and A. S. Taylor described their studies of the effect on the ionosphere of the annular solar eclipse of April 7. A portable ionosphere recorder was operated at Fort Clark, Texas, from April 5 to 15, and special measurements were made at Meadows, Maryland, on April 7. The results showed a definite drop of E-layer ionization approximately in phase with the eclipse and indicated a recombination coefficient of 10^{-8} or greater for the E layer. The decrease of F_2-layer ionization was not very definite and if present was small. The results indicated a recombination coefficient of 10^{-10} or less for the F_2 layer.

SAFETY RULES FOR ELECTRIC FENCES.

Electric fences that give a shock to an animal that touches them, are rapidly coming into use on the farms of this country. To prevent serious accidents, electric fences must be supplied with current from a properly designed and constructed controller.

Safety rules regulating the use of electric fences constitute Part 6 of the new edition of the National Electrical Safety Code. This is now available as Handbook H36. The regulations provide for the use of an adequate controller, for the marking of fences in public locations, and for the testing of equipment to see that it will fulfill the intended function.

Two States already regulate the sale and application of fence controllers, and others will probably follow their example, now that standard rules are available.

Copies of H36 are obtainable from the Superintendent of Documents, Government Printing Office, Washington, D. C., at 5 cents each.

ABSOLUTE STANDARD OF MUTUAL INDUCTANCE.

In the *Journal of Research* for June (RP1302), Chester Snow derives a formula for the mutual inductance of two helical wires wound bifilarly in a single layer upon a cylindrical form. Terms are retained which are of the order of magnitude (relative to the principal term) of the square of the ratio of the pitch of the windings to the radius of the cylinder. Account is taken of the diameters of the two wires (which need not be equal), the current distribution, the axial component of current and the azimuth of the two windings, and also of the fact that the mean cylindrical radii of the primary and secondary may not be exactly the same nor the turns of the one exactly midway between those of the other.

EQUIPMENT FOR AIR-CONDITIONING OF MATERIALS.

As part of an investigation of the drying of textiles, equipment was designed and built for the study of the moisture relations of materials in air of controlled relative humidities at elevated temperatures.

This equipment is described in detail in RP1303 by James G. Wiegerink in the June issue of the *Journal of Research*. The air is brought to the desired relative humidity and temperature by an arrangement including a saturator and automatically controlled heaters, and is then passed through a working chamber in which the material to be studied can be weighed rapidly and accurately. Provision is made for

measuring the rate of flow of air and for determining the relative humidity in the working chamber.

The equipment has been used at temperatures from 96° to 300° F., relative humidities from 10 to 90 per cent. for temperatures up to 212° F. and to the maximum relative humidities obtainable at atmospheric pressure for temperatures between 212° and 300° F. The temperature in the working chamber can be maintained constant within ±0.4° F. and the relative humidity within ±1 per cent. relative humidity for experimental runs of several days duration.

The equipment is of sufficient capacity for the study of ten different materials simultaneously. It is large enough to accommodate samples to be tested in various ways after drying, and to maintain a relatively large ratio of volume of air to volume of samples in order to insure uniform treatment through each sample and for the different samples. Provision is made for weighing the samples rapidly and conveniently while they are in the equipment at the desired relative humidity and temperature.

ACCELERATED WEATHERING OF TRANSPARENT PLASTICS.

In previous reports of the Bureau's work on transparent plastics it was noted that accelerated aging tests made with apparatus employing an inclosed carbon arc as the source of untraviolet light failed to correlate well with the results of outdoor exposure (*J. Research N. B. S.*, **19**, 367 (October, 1937), RP1031; and "Permanence of Plastics" in "Symposium on Plastics" published by American Society for Testing Materials, 260 South Broad St., Philadelphia, Pa.). A detailed investigation of accelerated weathering of transparent plastics was, therefore, undertaken in an endeavor to obtain a reliable test for evaluating the aging properties of new plastics and various compositions of previously employed plastics.

Sources of ultraviolet light which were studied included the Weather-Ometer, Fade-Ometer, Eveready Accelerated Testing Unit, General Electric Sunlamp with S-1 bulb, Cooper-Hewitt mercury arc lamp, Hanovia Alpine mercury arc lamp, and General Electric quartz-capillary mercury arc lamp with H-4 bulb. Many of these lamps were operated with and

without Corex D filters to determine their usefulness under these two conditions for making accelerated aging tests on transparent plastics. The effect of the temperature of the samples during irradiation was also investigated. The samples were exposed to the light with and without alternate cycles of wetting with water. Three different methods of wetting the sample were employed, namely, immersion in water, spraying with water, and exposure to a mist-laden atmosphere in a closed cabinet.

The effects of these different laboratory accelerated aging conditions upon various types of transparent plastics were evaluated by measurements of light transmission and haze, and by visual examination of the specimens for crazing, fusing, blooming, discoloration and warping. These results were compared with data obtained for specimens of the same plastics exposed to the weather in Washington. On the basis of these tests a method of accelerated aging of transparent plastics has been developed which gives results correlating well with actual behavior of the materials in outdoor exposure tests.

ABSORPTION OF WATER BY PLASTICS.

A method of measuring the absorption of water by plastics has been developed by Subcommittee V of Committee D-20 on Plastics of the American Society for Testing Materials, working in coöperation with subcommittees II and III of Committee D-9 on Electrical Insulating Materials. It is expected that this method will be adopted as a tentative standard at the 1940 annual meeting of the society in June. This method will replace the short-time water absorption tests provided in D48–37 for molded insulating materials, D229–38T for sheet and plate materials, D349–35T for laminated round rods, and D348–38T for laminated tubes.

Experimental data obtained in an exploratory investigation at the Bureau of the absorption of water by plastics in connection with the preparation of the above test method are summarized in a paper to be presented at the 1940 meeting. These data pertain to the effects of various conditioning procedures, times of immersion, and prior history of the specimens on the values observed for water absorption by

plastics. The plastics examined included phenol-formaldehyde molded, cast, and laminated materials, urea-formaldehyde molded and laminated materials, casein, cellulose nitrate, cellulose acetate, ethylcellulose, polyvinyl chloride-acetate, polyvinyl acetal, polystyrene in molded and cast forms, polymethyl methacrylate, and cold-molded plastics of the regular and phenolic types.

Data are also presented on the rates of absorption and desorption of water by these plastics for prolonged periods up to about 600 days. The variations in the dimensions of the specimens that are produced by these changes in moisture content are recorded.

SURFACE TREATMENT OF STEEL PRIOR TO PAINTING.

Surface treatment processes for application to plain and galvanized steel before painting, are described in Building Materials and Structures Report BMS44 which has just been released. The results of accelerated laboratory corrosion tests, designed to determine the relative effect of these treatments in improving the protective value of paints, are discussed. Comparison of results is limited to the tests themselves, and conclusions are drawn on the basis of factors affected by conditions peculiar to the individual test.

Among the treatments subjected to accelerated weathering, salt spray, and condensation corrosion tests, the hot-dip phosphate treatments for both plain and galvanized surfaces showed outstanding merit in improving the protective value of paints. Particularly effective protection was obtained when such treatments were used in combination with a primer of the inhibitive type. A phosphate cold wash for galvanized steel and two phosphate-chromate coldwash treatments for plain steel also appeared to improve paint protection.

The present report, copies of which are obtainable from the Superintendent of Documents, Government Printing Office, Washington, D. C., at 10 cents each, is the first of a series in which it is proposed to discuss the results of laboratory corrosion tests and supplementary outdoor exposure tests of surface treatments for the corrosion protection of plain and galvanized sheet steel.

THE FRANKLIN INSTITUTE

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, June 12, 1940.)

HALL OF THE COMMITTEE,
PHILADELPHIA, JUNE 12, 1940.

DR. FREDERIC PALMER, JR., in the Chair.

The following report was presented for final action:

No. 3071: Escape and Rescue from Disabled Submarines.

This report recommended the award of an Elliott Cresson Medal to the United States Navy, "In consideration of the development of means for saving human lives in disabled submarines and the successful application of the United States Lung and Rescue Chamber to this end."

At the March 13, 1940, Stated Meeting, the following report was presented for final action:

No. 3066: Stabilized Feedback Amplifier.

This report recommended the award of a John Price Wetherill Medal to Harold S Black, of New York City, New York, "In consideration of his work on amplifiers, especially his development of the negative feedback amplifier."

This recommendation was advertised in the May issue of the Journal of The Franklin Institute and the advertisement expired without the receipt of objection.

JOHN FRAZER,
Secretary to Committee.

LIBRARY NOTES.

The committee on Library desires to add to the collections of the Institute any technical writings of members who have had occasion to publish such material.

Literary contributions from author-members will be gratefully acknowledged, properly inscribed and noted in the Journal of the Institute.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The average cost for a print $8\frac{1}{2} \times 11$ inches is forty cents.

The library and reading room are open daily from nine o'clock A.M. until five o'clock P.M. Saturdays until 12 o'clock noon. 779 readers made use of the facilities during the twenty-six days of May.

RECENT ADDITIONS.

AERONAUTICS.

HARTNEY, HAROLD E. Up And At 'Em. 1940.

AGRICULTURE.

MATLIN, D. R. Growing Plants without Soil. 1939.

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BAMFORD, FRANK. Poisons. No date.
ELVEHJEM, C. A., AND P. W. WILSON, Editors. Respiratory Enzymes. 1939.
SHEPARD, HAROLD H. The Chemistry and Toxicology of Insecticides. 1939.

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American Institute of Chemical Engineers. Transactions 1939. Volume 35. 1940.
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GRADY, ROY I., JOHN W. CHITTUM AND OTHERS. The Chemist at Work. 1940.
MENDELSOHN, SIMON. Embalming Fluids. 1940.
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VAN NIEUWENBURG, C. J., AND G. DULFER. A Short Manual of Systematic Qualitative Analysis by Means of Modern Drop Reactions. Third Edition. 1938.

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Lincoln Electric Company. Procedure Handbook of Arc Welding Design and Practice. Sixth Edition. 1940.
MCEACHRON, K. B., AND KENNETH G. PATRICK. Playing with Lightning. 1940.
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BACHELIER, LOUIS. Les Nouvelles Méthodes du Calcul des Probabilités. 1939.
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SMITH, R. GOODWIN —. English Domestic Metalwork. 1937.

NAVAL ARCHITECTURE AND NAVIGATION.

BOWDITCH, NATHANIEL. American Practical Navigator. Revised Edition. 1939.

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MANN, W. B. The Cyclotron. 1940.

MILLER, ELMER S. Quantitative Biological Spectroscopy. 1939.

OLSON, HARRY F. Elements of Acoustical Engineering. 1940.

RAILROAD ENGINEERING.

BEAL, EDWARD. The Craft of Modelling Railways. 1938.

SCIENTIFIC ESSAYS.

SEDGWICK, W. T., AND H. W. TYLER. A Short History of Science. Revised by H. W. Tyler and R. P. Bigelow. 1939.

NOTES FROM THE BARTOL RESEARCH FOUNDATION.

A PORTABLE APPARATUS FOR RECORDING BURSTS OF COSMIC-RAY IONIZATION.*

BY

C. G. MONTGOMERY AND D. D. MONTGOMERY.

Although it is easy to detect and measure the bursts of cosmic-ray ionization, or Hoffmann Stösse, by means of vacuum tube electrometers [1] or electroscopes,[2] such arrangements are inconvenient to employ in field work since they require quite steady supports and relative freedom from vibration, as well as a degree of constancy of the potential sources not easily attainable. An arrangement which minimizes these difficulties is here described. The change in potential of the collecting electrode of the ionization chamber in which the burst occurs is amplified by a degenerative, directly-coupled vacuum tube amplifier, and the output pulse from this amplifier is applied to a "magic eye" indicator tube. This indicator tube acts as a cathode-ray oscillograph, and is directly photographed on continuously moving sensitive paper to give a record of the burst.

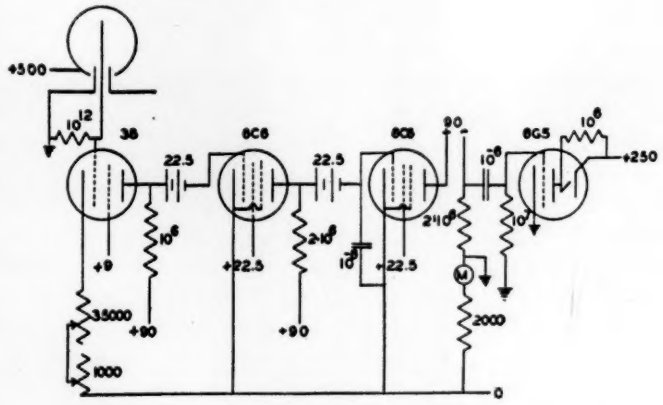
To achieve an amplification of a thousand-fold and at the same time have the action of the amplifier independent of changes in battery potential, temperature, and so forth, a directly coupled amplifier was employed, with a large negative feedback. Suppose we have an amplifier of gain g without feedback working into a resistance R, and suppose that a portion of this resistance is used to feed back the pulse to the input, then it is easy to show that the resultant gain, G, is given by $R/r[1 + (R + r)/rg]$. Now if g is large compared to R/r, the quantity in brackets in this expression becomes almost unity. The gain G is then determined only by the values of the resistances, and is independent of g. This re-

* Condensed from an article published in full in *Review of Scientific Instruments*, 10, 350 (1939).

moves any dependence on battery potentials, and gives a very stable arrangement. In the present amplifier $R/r = 2000$, and g is about 10^5. Thus a change in g by a factor as large as two changes G by less than two per cent. The circuit diagram of the amplifier is shown in Fig. 1.

The output of the amplifier is applied through a condenser to a 6U5/6G5 "magic eye" indicator tube which is used as an oscillograph. The circular fluorescent screen of this tube is sprayed by electrons, and so illuminated, except for a sector which is dark. The angle of this dark sector is determined

FIG. 1.



Circuit diagram of the apparatus. Resistances are indicated in ohms, capacities in farads.

by the potential applied to the tube. At the edges of the shadow there is a focusing action of the deflecting field which makes the illumination at the boundary considerably more intense than the rest of the lighted area. The boundary is thus very well defined and easy to photograph. An image of the fluorescent screen of unit magnification is cast upon the slit of a motor driven recording camera by a simple lens of two-inch focal length, stopped down to an aperture of approximately $f:5$. With Eastman recording paper number 2, a slit width of 0.01 inch, and a paper speed of five inches per hour, a record is obtained which is so sharp that it can be magnified about ten times for measurement without serious loss of definition. The image is quite intense, and much

higher recording speeds could be used without difficulty. The response of the 6U5/6G5 tube is not a linear one, a property which is very useful in its application to the measurement of bursts of cosmic-ray ionization. The nonlinearity of the response enables a high sensitivity to be used for the smaller, more frequent bursts, while the larger, and rarer, bursts do not give a deflection off the scale.

The volume in which the ionization is measured consists of that of three small spherical chambers connected in parallel. Each chamber is three inches in diameter and has $\frac{1}{16}$-inch thick iron walls. Each is constructed of two spun hemispherical shells brazed together. The electrodes are brass rods $\frac{1}{16}$ inch in diameter and two inches long mounted in plugs screwing into the bottoms of the spheres. The plugs are made of brass and paraffined hard rubber, and are furnished with guard rings. The leakage resistance attained in this way is greater than 10^{14} ohms. The spheres are mounted side by side on top of the box containing the amplifier and indicator tube, and contain argon or nitrogen at a pressure of 20 atmospheres. Immediately below the ionization chambers is a compartment into which a drying agent can be introduced to maintain the high degree of insulation necessary. The apparatus has been employed for observations in a mine as well as in the laboratory, and has been found to be very rugged and easy to use.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

The Distribution of Radiophosphorus and Radiogold in Rats after Oral, Intraperitoneal and Subcutaneous Administration.—J. O. ELY. So far as the author is aware the literature contains no information on the distribution of radioactive gold in the tissues after administration to rats. The distribution of radiophosphorus in the tissues of rats has been reported by Cohn and Greenberg (*Journal of Biological Chemistry*, **123**: 185, 1938) and by Chiewitz and Hevesy (*Nature*, **136**: 754, 1935). The results of some preliminary studies on the distribution of radioactive isotopes of phosphorus and gold in the tissues following administration to rats are presented here.

Radiophosphorus: Radioactive phosphorus was prepared by the bombardment of red phosphorus with deuterons in the cyclotron of the Biochemical Research Foundation of the Franklin Institute. From the radioactive element, sodium phosphate was then prepared by solution in HNO_3, decomposition of the excess acid, and neutralization with NaOH.

Measurements of the radioactivity of the solutions, using a suitable dilution and amount evaporated within an outlined area on a glass slide, were made with a Geiger-Mueller counter at a distance of 13 mm. from the counter tube. From the values obtained with the dilute solutions, the radioactivity of the solutions administered to the animals was calculated.

White rats were used. The tissues were removed and weighed immediately following decapitation of the animals. Samples of the tissues sufficiently small to make radioactive measurements of the total ash of the samples possible were ashed in small porcelain crucibles. The radioactivity of the ash was measured by placing the crucibles beneath the tube so that the bottoms were 23 mm. from the counter tube. The calculations were based on the number of counts produced per unit of time. From the values obtained for the small samples of tissue, calculations were made per gram of tissue.

The method was tested for accuracy by measuring the activity of a series of duplicate tissue samples of approximately equal weight. Table 1 shows the results obtained, the percentage difference being calculated on the basis of the lower of the two values obtained for the duplicate samples. The percentage difference represents the error for the whole procedure, including sampling, ashing, and measurement of the radioactivity.

TABLE 1.

The Percentage Difference in Measurements on Duplicate Samples of Radioactive Tissue.

Tissue.	% Difference.
Tibia	1.2
Skull bone	12.0
Spleen	3.0
Liver	6.0
Spleen (A different animal from Spleen No. 1)	11.0
Kidney	0.0

In Table 2 are shown the results of radioactivity measurements of various tissues. The values given in the table, expressed as percentage distribution of radioactivity per gram of tissue, were obtained by calculating first the number of counter impulses for 1 gram of tissue, and then calculating, for each tissue, the percentage of the total.

TABLE 2.

The Percentage Distribution of Radiophosphorus in Rats per Gram of Tissue.

Rat No. and Wt. (grams)	No. 1, 168.	No. 2, 190.	No. 3, 140.	No. 4, 140.	No. 5, 160.	No. 6, 129.	No. 8, 121.
Dosage (kicks per min./1000)	245	245	892	892	478.4	1,165	1,165
Quantity administered	1 cc.	1 cc.	1 cc.	1 cc.	1 cc.	0.5 cc.	0.5 cc.
Experimental period	73 hrs.	73 hrs.	23 hrs.	23 hrs.	43 hrs.	48 hrs.	7 days
Method of administration	Subcutan.	Orally	Orally	Intraper.	Orally	Intraper.	Intraper.
Tissue.	*Percentage Distribution of Radioactivity per Gram of Tissue.*						
Tibia	29.8	16.0	22.1	17.6	34.4	33.1	48.6
Thymus	11.0	12.4	10.7	10.7	9.7	26.3	11.5
Feces	9.9	3.3	17.8	10.2	13.2	—	—
Spleen	6.0	10.7	7.7	3.7	6.9	8.8	8.3
Kidney	8.3	10.9	6.7	6.5	5.4	7.8	8.3
Liver	7.7	10.7	7.8	9.9	6.5	8.4	7.4
Colon	9.9	9.0	7.2	13.1	5.1	6.6	5.3
Muscle (Gastrocnemius)	5.1	4.1	2.7	4.5	3.4	3.7	5.3
Lung	8.5	9.0	5.7	2.7	5.4	4.4	2.7
Testicle	—	—	—	—	1.8	—	—
Brain	1.3	2.2	1.0	0.9	0.9	0.9	2.5
Pancreas	2.5	11.7	5.4	11.3	3.6	—	—
Adrenals	—	—	5.3	4.5	3.6	—	—

It is of interest to note that per gram of tissue bone (tibia) contained the greatest amount, and brain the smallest amount of radioactivity. The thymus showed a relatively high value in all cases.

From the weights of the various organs and tissues and the radioactivity values found for them per gram of tissue, the total amount of radioactivity in some of the organs, the skeleton, and the musculature was calculated. These data are shown in Table 3. It was assumed that the tibia comprises 3.9 per cent. by weight of the skeleton (determined for

TABLE 3.

The Distribution of Radiophosphorus in Organs and Tissues of Rats.

Rat No. and Wt. (grams)	No. 1, 168.	No. 2, 164.	No. 3, 140.	No. 4, 140.	No. 5, 160.	No. 6, 129.	No. 8, 121.
Dosage (impulses per min.)/1000	245	245	892	892	478.4	1,165	1,165
Volume administered	1 cc.	1 cc.	1 cc.	1 cc.	1 cc.	0.5 cc.	0.5 cc.
Experimental period	73 hrs.	73 hrs.	23 hrs.	23 hrs.	43 hrs.	48 hrs.	7 days
Method of administration	Subcutan.	Orally	Orally	Intraper.	Orally	Intraper.	Intraper.
Tissue or Organ.	*Percentage Distribution in the Tissues or Organs.*						
Skeleton	37.7	30.6	54.5	41.5	57.9	44.2	49.2
Musculature	53.1	52.2	40.3	55.0	33.8	43.5	42.8
Liver	6.3	11.7	—	—	4.7	9.0	5.8
Spleen	0.6	1.4	1.0	0.8	1.0	0.6	0.6
Kidney (one)	0.5	0.9	1.3	0.8	0.5	0.6	0.6
Lung	1.2	1.8	1.3	0.5	0.4	0.8	0.4
Thymus	0.4	0.5	1.0	0.6	0.9	1.1	0.3
Brain	0.2	0.4	0.2	0.2	0.2	0.2	0.3
Pancreas	0.1	0.6	0.4	0.5	0.3	—	—
Adrenals (both)	—	—	0.06	0.1	0.04	—	—
Testicle (one)	—	—	—	—	0.3	—	—

a 154 gram rat) and that the musculature (including tendons and fat) comprises 47 per cent. of the weight of the animal (determined for a 190 gram rat). This value for the musculature was obtained by removing the skin and all viscera, and the brain, and subtracting from the weight in this condition the calculated weight of the skeletal structure, assuming that the tibia comprises 3.9 per cent. of the weight of the skeleton.

It may be seen in Table 3 that in all of the rats studied approximately 90 per cent. of the radiophosphorus was concentrated in the skeleton and the musculature. As shown in Table 2 the highest values per gram of tissue are shown by bone with muscle containing a far lower concentration. However, because of the large amount of muscle tissue compared to

skeletal tissue, the musculature contained, in general, approximately as much radiophosphorus as did the skeleton. The thymus, as an organ, contained a relatively small amount of the radiophosphorus, although per gram of tissue the value was relatively high.

In order to compare the relative absorption following oral and intraperitoneal administration rats No. 3 and No. 4, of equal weight, were given equal amounts of radiophosphorus, and the total radioactivity of organs and tissues (Table 3) was determined after a period of 23 hours. The total activity found for the tissues and organs studied from rat No. 3, which received the radiophosphorus orally, was equivalent to 51,158 counter impulses per minute; for rat No. 4, which received the radiophosphorus intraperitoneally, the value found was 79,781. The absorption by the organs and tissue was much greater following intraperitoneal administration than by oral administration.

A comparison of the absorption by the various organs and tissues (Table 3) following oral and subcutaneous administration was made with rats No. 1 and No. 2 which received the same amount of radiophosphorus and the tissues examined after the same interval (73 hours). The total value obtained for the organs and tissue studied of rat No. 1 (subcutaneous administration) was 69,872 counter impulses per minute, for rat No. 2 (oral administration) 36,075. Absorption was greater following subcutaneous administration than it was following oral administration.

Radiogold: Gold was made radioactive by bombardment of a small plate of gold with deuterons in the cyclotron. Gold chloride was prepared from filings from the bombarded surface of the gold plate by solution in hydrochloric and nitric acids and elimination of the excess nitric acid by boiling. The solution of gold chloride was prepared for administration to the animals by adjusting the pH to 6.8 with sodium hydroxide. The radioactivity of the solution was determined by the method described for the radiophosphorus experiments.

White rats were used. The technic of administration to the animals, the removal and ashing of the tissues, and the determination of the radioactivity was identical with that described for the radiophosphorus experiments. Small sam-

ples of tissues were used and from the values obtained the amount of radioactivity per gram of tissue was calculated. The gold chloride was toxic to the animals. A 2.0 per cent. solution was administered to rats No. 14 and No. 17; No. 14 died just prior to removal of the tissues.

TABLE 4.

The Distribution of Radiogold in Rats Following Oral, Intraperitoneal and Subcutaneous Administration.

Rat No., Wt. (grams), Sex	No. 1, 111, F.	No. 2, 138, F.	No. 14, M.	No. 17, 120, M.
Dosage (kicks per min./1000)	1300	126.6	5460	5460
Quantity administered	1 cc.	1 cc.	0.8 cc.	0.4 cc.
Experimental period	66 hrs.	22 hrs.	6 hrs.	46 hrs.
Method of administration	Orally	Orally	Intraper.	Subcutan.

Tissue.	*Distribution per Gram of Tissue.*							
	Impul. per Min.	%	Impul. per Min.	%	Impul. per Min.	%	Impul. per Min.	%
Kidney	887	43.2	107	2.0	1235	16.0	93	15.5
Adrenal	387	18.9	81	1.5	—	—	—	—
Liver	119	5.8	23	0.4	352	4.6	3	0.5
Feces	142	6.9	4952	91.1	—	—	174	29.0
Spleen	101	5.0	22	0.4	687	8.9	19	3.2
Thymus	95	4.6	17	0.3	—	—	5	0.8
Colon	93	4.5	26	0.5	—	—	—	—
Lung	85	4.1	166	3.1	1884	24.5	8	1.3
Pancreas	73	3.6	7	0.1	—	—	—	—
Femur	30	1.5	—	—	—	—	5	0.8
Blood (per cc.)	20	1.0	20	0.4	—	—	30	5.0
Muscle (Gastrocnemius)	16	0.8	7	0.1	—	—	7	1.2
Brain	5	0.2	10	0.2	—	—	1	0.02
Abdominal wall					1235	16.0	—	—
Abdominal wall (site of inject.)							8120*	—
Testicle					380	5.0	—	—
Large intestine and contents					680	8.8	—	—
Small intestine and contents					1242	16.1	4	0.7
Lymph node (left groin)							48	8.0
Lymph node (rt. groin, side of inject.)							180	31.5
Stomach tissue							3	0.6
Heart							5	0.8
Tail							6	1.0

* Not used in the calculation of percentage distribution.

The results are shown in Table 4 and are expressed per gram of tissue in two ways: (1) Counter impulses per minute, and (2) a percentage basis. It is apparent from Table 4 that there was a general distribution of the radiogold throughout all organs and tissues. Absorption, however, was apparently relatively slow as shown by the large amount still present in the feces in rat No. 2 after 22 hours and the large amount present at the site of injection in rat No. 17, after 46 hours. Apparently elimination of the radiogold is by both the urinary

and intestinal tract regardless of the method of administration as shown by the relatively high values shown by the kidney and portions of the intestinal tract.

Summary: Data on the distribution of radiophosphorus and radiogold in the tissues following oral, intraperitoneal, and subcutaneous administration have been presented. The skeletal and muscular systems accumulated about 90 per cent. of the administered radiophosphorus. Radiogold, as the chloride, was absorbed relatively slowly and accumulated in all tissues. Radiogold is eliminated apparently, by both the urinary and alimentary systems.

Systematic Qualitative Organic Microanalysis: Determination of the Refractive Index of Liquids.—HERBERT K. ALBER and J. T. BRYANT (*Ind. Eng. Chem., Anal. Edition*, **12**: 305, 1940). In the scheme of qualitative organic microanalysis which was outlined (JOURN. FRANKLIN INSTITUTE, **226**: 813, 1938) one of the physical constants necessary for the identification of unknown liquid samples is the refractive index. Several methods for the microdetermination of this physical constant have been tested on a comparative basis, e.g., the Abbé-refractometer, dipping refractometer with auxiliary prism by Pregl, the *schlieren* method of Emich and co-workers, the Becke-line method using glass powders of known refractive indices as recommended by Kofler, procedures by Wright, and the microrefractometer of Edwards and Otto which is a modification of Jelley's instrument. These methods did not meet all of the requirements for the systematic scheme which may be summarized as follows: accuracy within ± 0.001; maximum amount of liquid to be used = 10 cu. mm., which should be recoverable unchanged after the determination; time limit for one determination = 10 minutes; and a total range of refractive indices from 1.33 to about 2.00.

The most suitable procedure was found to be that of L. Nichols (*National Paint Bulletin*, **1**: 5 (Jan.), 12 (Feb.), 14 (Mar.), 1937). Based on the macro instrument with a capacity of from 100 to 200 cu. mm., a microrefractometer with a capacity from 5 to 7 cu. mm. was constructed in cooperation with Mr. Nichols. The instrument consists of two

tiny glass prisms of the same refractive index, cemented into a metal ring on a glass slide. Across the base of the prisms is drawn a fine line. The cell is filled with the liquid to be tested or a standard liquid of known refractive index, by means of a capillary pipet, and covered with a cover glass. When the cell is placed on the stage of a microscope, magnification from ×80 to ×140, the light strikes the fine line, passes undeviated through the prisms, and on entering the liquid is refracted, forming two separated lines. The distance between the latter is directly proportional to the difference in the refractive indices between glass prisms and liquid and is measured by means of a finely calibrated ocular micrometer, or filar micrometer. Using the same optical system, including constant microscope tube length, a graph (or table) is prepared from the results of five or six experiments carried out with standard liquids of known refractive index at the same temperature; in the graph a straight line is obtained which correlates the number of divisions observed with refractive indices. After each measurement most of the liquid, about 80 per cent. or more, can be recovered by means of a capillary pipet and can be used for any other test of the qualitative scheme. The cell is then carefully cleaned, and is ready for another determination.

The refractive indices of more than 100 compounds of varying physical properties have been measured in the microrefractometer. The accuracy determined in comparison with an Abbé-refractometer (accuracy = ± 0.0001) was found to be at least within ± 0.001. In this investigation special emphasis has been placed on the construction of a simple instrument suitable for the scheme of qualitative analysis. The accuracy was increased to within ± 0.0005 by careful control of temperature and wave-length of light by means of a metal water jacket and colored filters respectively.

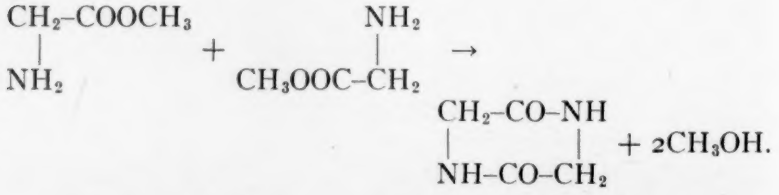
The microrefractometer has two cells, the one containing prisms of refractive index = 1.72, the other = 1.52. This allows a wider range of refractive indices; the cell 1.52 is preferable for measuring refractive indices below 1.40 and above 1.65, the cell 1.72 for refractive indices between 1.40 and 1.65, and from 1.85 to 2.00 or higher. The accuracy of the results obtained was the same with both cells.

The microrefractometer may be used also in the determination of refractive indices of *solid* compounds. After carrying out the Becke-line test with mixtures of reference liquids, one component of which may be volatile, in many cases as little as 10 to 20 cu. mm. of the immersion liquid remained; the refractive index of this liquid was then determined in the microrefractometer. Satisfactory results were obtained, since the accuracy (± 0.001) is the same in both methods.

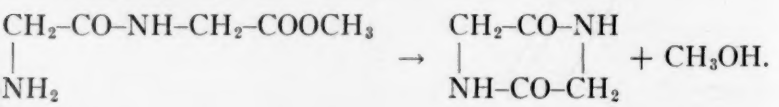
Seminar on Polycondensation of the Peptide Esters of Glycine.—At a seminar of the Biochemical Research Foundation on April 2nd, DR. EUGENE PACSU of the Department of Chemistry, Princeton University, told of his researches on "Polycondensation of the Peptide Esters of Glycine." He first became interested in this subject through Wrinch's "cyclol" theory of protein structure. Knowing that esters of glycine and glycine peptides condense with liberation of alcohol, he thought it might be possible to form a cycle of six glycine residues and in this way give substantiation to the "cyclol-6" structure postulated by Wrinch.

Dr. Pacsu classified all condensations of the peptide esters in four distinct types:

1. The mono-peptide ester, methyl glycine, condenses readily, splitting out two molecules of methyl alcohol from two molecules of the ester. Diketopiperazine is formed, thus:



2. The dipeptide ester, methyl glycyl glycine, splits out one molecule of methyl alcohol to form the same compound, diketopiperazine, thus:



3. With the tripeptide ester, two molecules lose one molecule of methyl alcohol intramolecularly, and a hexapeptide ester is formed:

$$\begin{array}{l} CH_2\text{–}CO\text{–}NH\text{–}CH_2\text{–}CO\text{–}NH\text{–}CH_2COOCH_3 \\ | \\ NH_2 \quad + \end{array} \rightarrow$$

$$\begin{array}{l} CH_3OOC\text{–}CH_2\text{–}NH\text{–}CO\text{–}CH_2\text{–}NH\text{–}CO\text{–}CH_2\text{–}NH_2 \\ \quad CH_2\text{–}CO\text{–}NH\text{–}CH_2\text{–}CO\text{–}NH\text{–}CH_2\text{–}COOCH_3 \\ \quad | \\ \quad NH \qquad\qquad\qquad\qquad\qquad\qquad + CH_3OH. \\ \quad | \\ \quad CO\text{–}CH_2\text{–}NH\text{–}CO\text{–}CH_2\text{–}NH\text{–}CO\text{–}CH_2\text{–}NH_2 \end{array}$$

4. A tetrapeptide ester undergoes no condensation whatsoever.

The experiment was reported of heating the hexapeptide ester at a constant temperature (102° ± 1°). Condensation occurred, but this was of the type characteristic of the tripeptide ester. There was no cyclization as shown by methoxyl determinations. A series of bimolecular reactions took place, yielding the 12-, 24-, 48-, and 96-peptide (3×2^n peptide) esters of glycine. The final 96-peptide ester had a methoxyl content of 0.56 per cent., the theoretical content for the methyl ester of the 96-peptide. This would have been zero if cyclization had occurred.

At a higher temperature the condensation rate was faster. When the hexapeptide ester was heated at 130° ± 1° for 144 hours, the methoxyl content was 0.58 per cent., indicating the presence of the 96-peptide ester. Heating for 36 days further at this temperature reduced the methoxyl content only to 0.54 per cent., indicating that the condensation stops at the 96-peptide ester, and that there was no cyclization. The same result was obtained when the 3-peptide or the 12-peptide ester was heated at constant temperature. In every case the final product was the 96-peptide ester as indicated by the methoxyl content.

It was interesting to note that the 1-, 2-, 3-, and 6-peptide esters will condense, but that the 4- and 5- will not. It was

also not possible to form the 9-peptide ester, either by condensing the 6-peptide with the 3-peptide or by condensing 3 molecules of the 3-peptide. The composition of the condensation products formed at elevated temperature from the 3-peptide ester was determined at certain intervals of time by separation of the constituents with methyl alcohol and hot water. Only tri-, hexa-, dodeca- and higher peptide esters were found to be present.

Dr. Pacsu suggested that this procedure of condensation may go on *in vivo* by the action of enzymes, building up the amino acids into units of 96. He also drew attention to the fact that this unit of 96 amino acid groups was exactly 1/3 of 288 which has been suggested by Bergmann and Niemann as the number of amino acid residues in the Svedberg protein unit of 35,000 molecular weight. The extreme insolubility of the 96-glycine ester was suggested as being due to the absence of a side-chain in the glycine molecule. If, however, longer chain amino acids are condensed, the peptide esters show much greater solubility than the corresponding glycine peptides. The polypeptide of *d*(−)-glutamic acid, for example, is very soluble. This had been found by Bruckner and Ivanovics as a soluble natural polypeptide produced by *B. mesentericus* and *B. anthracis*. Further study on peptides of this type Dr. Pacsu thought might result in the production of water soluble substrates of known structure suitable for enzymatic investigation.

(Reported by Gladys E. Woodward)

BOOK REVIEWS.

THE PHILOSOPHY OF PHYSICAL SCIENCE, by Sir Arthur Eddington. 230 pages, 15 × 22 cms. New York, The Macmillan Company, 1939. Price $2.50.

The author of this book is well known to a host of readers and thinkers along philosophical lines throughout the world. He is quoted equally as much, if not more than others for his presentations of basic modern thought and opinions, particularly among the lay intelligentsia. There is, therefore, a certain expectation before reading this book, that it leads into worthwhile channels.

The contents of this book represents the substance of the course of lectures delivered by the author as Tarner Lecturer of Trinity College, Cambridge 1938. The plan of bringing into view the subject matter gives evidence of careful consideration. At the outset it treats with scientific existemology where existemology is defined as that branch of philosophy which treats of the nature of knowledge, and scientific existemology as the sub-branch of existemology which deals with the nature of this part of our knowledge and therefore indirectly with the nature and status of the physical universe to which it formally relates. This gives the tenor of what follows. Examined are the relativity and quantum theories, the concepts of analysis, structure and existence, some very strong and courageous statements on the physical universe, and the development of an outline of a general philosophical outlook which a scientist can accept without inconsistency. To those not directly associated with the broadness of science as here sketched, the book brings a sense of humility and awe of the vastness of the universe, and to those who are more familiar with science generally it brings a recognition to a degree at least of how little is now known and how insecure such knowledge is. The book contains views to which some may not agree but it does set the mind to work and is worthwhile reading.

R. H. OPPERMANN.

DÉCHARGE ÉLECTRIQUE DANS LES GAZ, par Marcel Laporte. 222 pages, illustrations, 11 × 17 cms. Paris, Armand Colin, 1939. Price 17 fr.

Since the middle of the nineteenth century the phenomena associated with electric discharges in gases have been the subject of numerous investigations, the results of which are of great theoretical and technical importance. To these investigations we owe not only fundamental advances in our knowledge of the constitution of the atom and of the reactions of matter and radiation, but also those applications which, beginning with the utilization of x-rays, are now being extended to the fields of lighting and of electrical engineering, where we may confidently expect some far-reaching developments.

French science, which figured brilliantly at the beginning of this research, has during the last few years kept somewhat aloof, and M. Laporte now fills a lack in the French language on a subject to which he has made a significant personal contribution. He here sets forth, with precision but with a minimum of mathematics, those findings which are most necessary to an understanding of

the various modes of electric discharge in gases and the attending luminous phenomena, providing in this small volume from the Collection Armand Colin a guide for students and engineers as well as for the layman who wishes to become acquainted with one of the most active branches of modern physics.

E. E.

THE PHYSICAL EXAMINATION OF METALS, by Bruce Chalmers. Volume I, Optical Methods. 181 pages, illustrations, 15 × 22 cms. New York, Longmans, Green & Co., 1939. Price $4.00.

There are certain methods for the physical examination of metals that are perhaps looked upon with uncertainty by many metallurgists, especially those who have had little training in physics. While the main properties of light have been put to use in the examination of metals, an understanding of the wave nature of light and associated properties will lead to methods of technique and application toward a more sound knowledge of metals. The author attempts in this book to explain some of the physical theory, the more important applications that have been made, and to describe technique in some instances. It is volume one of a two volume set, dealing entirely with optical methods. The second volume, yet to appear, will be on other branches of physics as applied to the investigation of metals.

A brief account of the properties of light is first given as a foundation for the treatment of geometrical optics which goes into the practical almost immediately with examples of the law of reflection in the optical lever. Properties of interference and diffraction are next treated and some applications touched on here are the measurement of elastic moduli, plane bending apparatus, measurement of size of powders and ultra-violet microscopy. Under polarized light are found investigation of surface changes, relation between optical properties and composition, and determination of micro-structure, state of strain, presence and nature of inclusions. The concluding chapter of the book examines briefly the process by which light is produced and the relationship to optical methods.

The book is not by any means a complete treatment but there is plain evidence of careful selection of channels for most profitable pursuit. A feature is the follow through of the subjects discussed to the point of revealing information on the present known state of results obtained and a frank statement regarding accuracy and dependability. The book is practical and should be of value to metallurgists.

R. H. OPPERMANN.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

Report No. 681, The Unsteady Lift of A Wing of Finite Aspect Ration, by Robert T. Jones. 8 pages, illustrations, 23 × 29 cms. Washington, Government Printing Office, 1940. Price 10 cents.

Unsteady-lift functions for wings of finite aspect ratio have been calculated by correcting the aerodynamic inertia and the angle of attack of the infinite wing. The calculations are based on the operational method.

The starting lift of the finite wing is found to be only slightly less than that of the infinite wing; whereas the final lift may be considerably less. The theory indicates that the initial distribution of lift is similar to the final distribution.

Curves showing the variation of lift after a sudden unit change in angle of

attack, during penetration of a sharp-edge gust, and during a continuous oscillation are given. Operational equivalents of these functions have been devised to facilitate the calculation of lift under various conditions of motion. As an application of these formulas, the vertical acceleration of a loaded wing caused by penetrating a gust has been calculated.

Report No. 682, Flame Speeds and Energy Considerations for Explosions in a Spherical Bomb, by Ernest F. Fiock, Charles F. Marvin, Jr., Frank R. Caldwell, and Carl H. Roeder. 20 pages, illustrations, 23 × 29 cms. Washington, Government Printing Office, 1940. Price 10 cents.

Simultaneous measurements have been made of the speed of flame and the rise in pressure during explosions of mixtures of carbon monoxide, normal heptane, iso-octane, and benzene in a 10-inch spherical bomb with central ignition. From these records, fundamental properties of the explosive mixtures, which are independent of the apparatus, have been computed. The transformation velocity, or speed at which flame advances into and transforms the explosive mixture, increases with both the temperature and the pressure of the unburned gas. The rise in pressure has been correlated with the mass of charge inflamed to show the course of the energy development.

Comparable mixtures of the three hydrocarbon fuels expanded about the same amount upon burning and therefore developed about the same power, despite differences in the rate of burning. The addition of ethyl fluid produced no measurable change in flame speed over the range of conditions studied. None of the characteristics of normal burning seems to give a clue as to the relative tendency of fuels to knock. The observed values of pressure are lower than these calculated on the assumption that reaction goes to equilibrium within a very short distance behind the flame front. This fact, together with other independent evidence, points to a continued evolution of energy within gas that has already been traversed by flame.

Report No. 686, Stability of Castering Wheels for Aircraft Landing Gears, by Arthur Kantrowitz. 16 pages, illustrations, 23 × 29 cms. Washington, Government Printing Office, 1940. Price 10 cents.

A theoretical study has been made of the shimmy of castering wheels. The theory is based on the discovery of a phenomenon called kinematic shimmy. Experimental checks, use being made of a model having low-pressure tires, are reported and the applicability of the results to full scale is discussed. Theoretical methods of estimating the spindle viscous damping and the spindle solid friction necessary to avoid shimmy are given. A new method of avoiding shimmy—lateral freedom—is introduced.

LES NOUVELLES MÉTHODES DU CALCUL DES PROBABILITÉS, par Louis Bachelier. 69 pages, 17 × 29 cms. Paris, Gauthier-Villars, 1939. Price 25 fr.

M. Bachelier, since 1927 Professor of the Differential and Integral Calculus at the University of Besancon, is well known for his numerous contributions to the calculus of probability and to the laws of chance. He now gives us a description of his use of certain new concepts which led to a study of questions so complex that our former methods would have been quite inadequate.

Though the discovery of the asymptotic formulae marked the greatest advance ever made in the calculus of probability and though the laws, which are expressed in these and which we owe to De Moivre and to Laplace, comprise what is of real importance in this analysis, yet the knowledge based on these laws and the corresponding theories remained static for almost a century, because the concepts and the means employed permitted the solution of only simple problems. By considering problems calling for new concepts, the author has changed the level of our thinking, adding whole chapters to the calculus of probabilities and bringing simplifications to classical problems.

The asymptotic formulae are, so-to-speak, continuous in space; those which are also continuous in time may be termed hyperasymptotic, the latter being exact while the former are approximate. The concept introducing the idea of time assimilates the transformations of probabilities to physical phenomena and gives vital form to a most abstract science. By considering only general theories and by omiting all demonstration and giving merely essential conclusions, the author has been able to condense the hundreds of pages that a complete development would require into the small compass of this work.

E. E.

SEMI-CONDUCTORS AND METALS, by A. H. Wilson. 119 pages, illustrations, tables, 15 × 22 cms. New York, The Macmillan Company, 1939. Price $2.00.

From time to time in these pages mention is made of accounts on various subjects known as the Cambridge Physical Tracts. This book is one of this series of accounts with the aim of simplifying some of the main achievements of the theory of metals in the last ten years.

It begins with a treatment of fundamental ideas which are necessary for a rough qualitative survey of the subject and proceeds into the energy levels of a three dimensional crystal giving general theory required for discussions of the finer details. The remaining chapters are devoted to the applications of the results. They cover specifically the structure of metals, semi-conductors, the thermal and magnetic properties of metals and conductivity.

The author realized the difficulty of treating so complex a subject in a simplified manner and to supplement this work a list of references is given for fuller understanding. Those who are familiar with the status of the subject will find this work valuable.

R. H. OPPERMANN.

A SHORT HISTORY OF SCIENCE, by W. T. Sedgwick and H. W. Tyler. Revised by H. W. Tyler and R. P. Bigelow. 512 pages, plates, illustrations, 15 × 22 cms. New York, The Macmillan Company, 1939. Price $3.75.

A knowledge of the history of science is of as great a value to the layman today as knowledge of political and economic history, for science plays a part of equal importance and is destined to at least continue in this position. For men engaged in scientific pursuits or in the application of science, a background of history is necessary to gain greater appreciation of the subject. Therefore, when a book on the subject makes its appearance, curiosity is aroused with regard to the extent of its coverage as well as its method of presentation.

This book is an outgrowth of a course of lectures at the Massachusetts Institute of Technology. As such it must be assumed that it primarily is for men of science and technology, but on examination it is evident that its usefulness is of a broader scope, to include readers of equal educational attainments in other fields. There is little in the book not readily understood by the general reader.

The history begins with the dawn of civilization by giving the best speculations of the origin of man and the crude and primitive methods in his struggle for existence. Then historical periods of Egypt and the East are outlined. From the beginnings of science in the East the story proceeds through the earliest Greek science, science in the Golden Age of Greece, science in Alexandria, the Roman world, Hindu and Arabian science and the progress of science to A.D. 1450. The influence of the Christian Church shortly after this time, it is recorded, was of importance in the general development of intellectual life. Interest in astronomy and natural science was inspired. Later there came mathematics and mechanics in the sixteenth centuries and natural and physical science of the seventeenth century.

The division of modern science in the book is entered in the seventeenth century with the invention of analytic geometry by Descartes. This is followed by ideas regarding heat, light, electricity, botany, medicine etc. of the eighteenth century, and physical and chemical sciences of the nineteenth century. The treatment ends with the dawn of the twentieth century.

The editors explain in the preface that no attempt is made to bring the history up to date in such matters as the new mathematical physics and the advances in the chemical and biological sciences characteristic of the twentieth century, since the available literature is abundant, and it may be doubted if they are not too close to our generation for a just historical perspective.

The treatment is a good history of foundations upon which recent as well as earlier advances were based. Emphasized is the evolution of scientific methods. It is an adequate, colorful presentation of this important chapter in the history of human culture. The coverage is some 512 pages including appendixes and name index.

R. H. OPPERMANN.

MATHEMATICAL METHODS IN ENGINEERING, by Theodore v. Karman and Maurice A. Biot. 505 pages, illustrations, 16 × 24 cms. New York, McGraw-Hill Book Company, Inc., 1940. Price $4.00.

There are many situations in the practice of engineering where mathematical methods are more or less cut and dried. Both in construction and operation and in many cases even in analytical work the limits of the application of mathematics are often narrow, being the mere working out of set formulas. The danger here to the individual engineer is very apparent. This book has for its purpose a demonstration of the mathematical approach to engineering problems which should go far to furnishing the proper mathematical understanding of these problems, hence avoiding the rut that is possible in routine engineering work and laying the groundwork for accurate calculations in a shorter time as well as preparing for the unusual and unexpected.

The prerequisites for the study of the subject are ordinary calculus, the elements of analytical geometry and the elements of algebra, which include linear equations, determinants, and some knowledge of algebraic equations. The

treatment starts with a short review of the fundamental theorems on differential equations of one independent variable. Then consideration is given to linear differential equations, especially to equations with constant coefficients and their special methods of solution. A treatment on Bessel functions completes the part of the book which is fundamentally of a mathematical character.

Henceforth the work deals with engineering subjects, with a filler between them of necessary information of a mathematical nature. Concepts of and problems in dynamics, small oscillations of conservative and non-conservative systems, the differential equations of the theory of structures, Fourier series applied to structural problems, complex representation of periodic phenomena, transient phenomena, and equations with finite differences applied to engineering are subsequent divisions of the book.

The problems for exercise at the end of each chapter contain, besides exercises in mathematical operations, additional applications to various fields of engineering such as fluid mechanics, heat conduction etc. Answers can be found in the back of the book. There is one small error, perhaps a misprint, of little consequence in the table of contents. The first part of Chapter V is listed as on page 152. It should be page 162.

To compile a useful book of this kind for the practicing engineer comprises a difficulty of which the authors are well aware. It must be remembered that the variety of problems and methods encountered in practice are wellnigh without limit. Nevertheless, the book is a strong effort in this direction on the subjects covered and it is well worth examination by mathematicians, instructors, students and practicing engineers.

R. H. OPPERMANN.

GENERAL PHYSICS FOR STUDENTS OF SCIENCE, by Robert Bruce Lindsay. 534 pages, tables, illustrations, 16 × 24 cms. New York, John Wiley & Sons, Inc., 1940. Price $3.75.

The title of this book indicates that it is intended to fit into a particular niche in the college educational system. Later, in the Preface, the author states that it is the purpose of the book "to serve as a basic introductory textbook for science students who have had mathematics through elementary calculus, and also to provide an intermediate and more rigorous course for such students as have already taken an elementary, descriptive course in physics." That there are situations in the textbook field for such a bridge there is little doubt, and at the same time the book may have possibilities in reaching many readers other than students of a prescribed course, having sufficient prerequisites and needing much of the contents.

The construction of the book reflects the usual logical sequence of subjects. Only here and there may be noted slight digressions from this trend. This can be seen from the coverage of the larger divisions of which there are five. A general introduction is followed by treatments on mechanics, heat, electricity and magnetism, and radiation. Also from this may be seen what is one of the best features of the book—the manner of presentation. For instance, the general introduction does not give a rigorous review of the last parts of prerequisite subjects, a statement or two regarding the coverage of the book, an abstract of the subject, or even the fundamentals upon which the rest of the work is based. Instead, there is an admirable approach from the "human interest" angle, almost

wholly of a non-mathematical nature, in a conversational tone using everyday language and making analogies within the general experience of everyone. Discussed are the nature of physics, the history of physics and some properties of matter. This type of introduction goes far in not only giving a clear background, but in instilling a feeling of confidence resulting in a desire to know more. It sets a standard for the rest of the book as well as leaving openings for the injection of this type of stimulant whenever the treatment in later divisions requires it.

Perhaps the next best illustration of the manner of presentation is in part five. This is devoted to radiation and covers the subjects, in order, of wave motion and acoustics, light waves and rays, physical optics, and electromagnetic radiation and the constitution of matter. Under the latter subject is a brief treatment on the theory of relativity. One would question the advisability of including this in a book on fundamentals but this and similar brief discussions of somewhat advanced subjects are here, and are covered with the proper approach and of sufficient extent as to give an insight without too much detail.

At the end of each chapter are thought-provoking exercises. A subject index appears in the back which adds to the value of the book for reference purposes. The book is one of the best of the modern works devoted to the fundamentals of general physics. It lays down a philosophy which seeds satisfaction from within and it prepares the way for further study.

R. H. OPPERMANN.

THE ULTRACENTRIFUGE, by The Svedberg and Kai O. Pedersen. 478 pages, plates, illustrations, tables, 17 × 25 cms. Oxford, Clarendon Press, 1940. Price $12.50.

Sometime ago there was mention made in these pages of a book in German on this subject. That book was much shorter than the one at hand although it treated the subject in a masterly fashion. The present book is in the form of a monograph, a summation of the work to date in this most promising field. It is a story of subjecting materials to another extreme condition, the methods employed, and the results observed.

The work starts out by giving an explanation first of elementary theory, then the detailed theory of sedimentation. Here it is shown that the movement of molecules or small particles suspended in a solution exposed to the action of a centrifugal field is determined not only by their sedimentation but also by their Brownian movement due to molecular bombardment. The next part of the book deals with the construction and operation of ultracentrifuges and is conveniently divided into, first, a short explanation of principles covering the cell and column of solution, the rotor, bearings, drive etc., and, second, the details including the high speed ultracentrifuge with oil turbine drive and then with air turbine drive. These sections are especially interesting in the historical information given, in that the difficulties and aims in development are given, revealing much of value in the way that time and costly experimental work may be eliminated by those who desire to work along these lines.

Part three of the book is devoted to methods of measurement. This gives a chronological outline of the different investigations carried out with the ultracentrifuge, the type of centrifuge used, a description of the changes introduced in the experimental technique from time to time, and some examples of the ultracentrifugal investigations. Then there follows a survey of the results obtained

with the ultracentrifuge. Both the work with proteins and with organic colloids (except proteins) are covered. The authors state in the preface "unfortunately it has not been found possible, as we had hoped, to review the experimental data obtained with the air-driven ultracentrifuges, but references are given in the bibliography at the end of the book." The bibliography referred to is quite complete and conveniently arranged. Following this there is a subject and author index.

The book is a monumental work, for which there is immediate need. It is the result of a tremendous amount of search and analysis of the literature coupled with expert knowledge and much experience in the field. It will go far toward extending research and development.

R. H. OPPERMANN.

PUBLICATIONS RECEIVED.

Magnetism and Very Low Temperatures, by H. B. G. Casimir. 93 pages, illustrations, 14 × 22 cms. Cambridge, University Press, 1940. Price $1.40.

Natural Color Processes, by Carlton E. Dunn. Third Edition, 232 pages, plates, 14 × 21 cms. Boston, American Photographic Publishing Company, 1940. Price $2.00.

Principles of Alternating-Current Machinery, by Ralph R. Lawrence. Third Edition, 678 pages, illustrations, 16 × 24 cms. New York, McGraw-Hill Book Company, Inc., 1940. Price $5.50.

National Research Council. Twelfth Report of the Committee on Catalysis. 388 pages, illustrations, 15 × 24 cms. New York, John Wiley & Sons, Inc., 1940. Price $5.00.

Conversion of Petroleum Production of Motor Fuels by Thermal and Catalytic Processes, by A. N. Sachanen. 413 pages, illustrations, tables, 16 × 24 cms. New York, Reinhold Publishing Corporation, 1940. Price $6.00.

Electrochemistry and Electrochemical Analysis, Volume II, Gravimetric Electrolytic Analysis and Electrolytic Marsh Tests, by Henry J. S. Sand. 149 pages, illustrations, 13 × 19 cms. London, Blackie & Son Limited. Price 5 s.

Elements of Strength of Materials, by S. Timoshenko and Gleason H. MacCullough. Second Edition, 365 pages, illustrations, 16 × 24 cms. New York, D. Van Nostrand Company, Inc. Price $3.25.

Canada, Department of Mines and Resources. Investigations in Ore Dressing and Metallurgy, July to December, 1938. 132 pages, 17 × 25 cms. Ottawa, King's Printer, 1940. Price 50 cents.

National Advisory Committee for Aeronautics. Technical Notes: No. 759, Pressure-Distribution Investigation of an N.A.C.A. 0009 Airfoil with a 30-Percent-Chord Plain Flap and Three Tabs, by Milton B. Ames, Jr., and Richard I. Sears. 43 pages, illustrations. No. 761, Pressure-Distribution Investigation of an N.A.C.A. 0009 Airfoil with an 80-Percent-Chord Plain Flap and Three Tabs, by Milton B. Ames, Jr., and Richard I. Sears. 34 pages, illustrations. No. 762. The Flow of a Compressible Fluid Past a Sphere, by Carl Kaplan. 15 pages, illustrations. No. 763, Wind-Tunnel Investigation of Two Airfoils with 25-Percent-Chord Gwinn and Plain Flaps, by Milton B. Ames, Jr. 13 pages, illustrations. 4 pamphlets, 20 × 26 cms. Washington, Committee, 1940.

CURRENT TOPICS.

85 Miles of Solid Rock.—(*Refrigerating Engineering*, Vol. 39, No. 4.) When the right of the city of New York to take Delaware River water went through the courts somebody must have raised the objection that a tunnel 85 miles long would be necessary to get from the river to the city. But time alone told that most of this 85 miles was through solid rock. The question came up back in 1926 and it took five years to get the legal rights straightened out. Then it took five years to get the money—170 million to dig this long hole alone. Now the digging is well under way and the tube will carry water to the city in two years—440 million gallons daily. A tube this long and 13 ft. in diameter is something to contemplate. It is built in sections between vertical shafts. There are 33 shafts, so it is around three miles between any two of them on the average. They are deep, running from 500 to 1000 ft. down from the surface, and as much as 650 ft. below sea level in absolute elevation. At this depth one cares little what there is above. There is no danger of running into sewers or cemeteries and what difference do a few rivers make? So we find them going under the Hudson in the neighborhood of Beacon, N. Y., and also going right under the Kensico reservoir. At this point the present work stops. They already have a 20 ft. tunnel from there to the city. The origin of the thing is at Lackawack, N. Y., attached to a stream feeding the Delaware. From each end of each section they blast away two to three times daily and make 10 or more ft. of headway each time. There are 11 separate tunneling contracts let. It was a problem to find places at the surface for depositing the rock removed. Land had to be bought, some of it previously growing perfectly good crops.

R. H. O.

Alkali Slurry for CO_2 Recovery.—GUSTAVE T. REICH of the Pennsylvania Sugar Co., Philadelphia, Pa., in *Chemical and Metallurgical Engineering*, Vol. 47, No. 3, gives interesting possibilities for greatly increasing the efficiency of carbon dioxide recovery from flue gases for use in the manufacture of liquid and solid carbon dioxide. The process described is aimed at overcoming the difficulties of the standard absorption process and at the same time to utilize alkali carbonate as the absorption medium. Such an absorbent, besides being inexpensive, imparts no taste or odor to the

CO_2, is not volatile or flammable, and hence is suitable for use in plants where these characteristics for an absorption material are very desirable. The essential and characteristic feature of the process consists in carbonating a saturated solution of alkali bicarbonate and alkali carbonate to produce a solution containing solid alkali bicarbonate in suspension. The suspended solid bicarbonate is then dissociated to alkali carbonate and CO_2 by means of heat, leaving a solution still saturated with bicarbonate. This permits the recovery of CO_2 more efficiently, with lower heat expenditure and smaller heating surface than by the standard absorption process. The use of the slurry of solid bicarbonate overcomes the mechanical and expensive disadvantage of handling the solid and permits pumping and early dissociation of the suspended material. The slurry consists of approximately 10 lb. of sodium carbonate and 50 lb. of sodium bicarbonate per 100 lb. of water. Based on a production of 2,000 lb. CO_2 per hour from flue gas containing 14 per cent. CO_2, this process removes all but 2 per cent. CO_2 as compared with 8.4 per cent. by the standard absorption process. Application of the process may also be made in the alkali industry, using highly concentrated slurry, producing carbonate and simultaneously liberating a 99.8 per cent. CO_2. Further, there may be a field in the chemical industry where the presence of CO_2 interferes with smooth operation.

R. H. O.

Giants Ahead?—(*Industrial Bulletin* of Arthur D. Little, Inc., No. 156.) One of the trends shaping the future of the still unstable American "race" is the tendency toward greater height with each succeeding generation. Examination of statistics that go back for about a hundred years reveals interesting information. A study of Harvard students whose ancestors had been in America for at least two generations indicates a gradual and regular increase of one inch per generation for four generations. Similar figures have been noted for girls, at Wellesley and other colleges. Jackson at Minnesota observed, "The younger college students represent a group precocious physically as well as mentally." It appears to be established that college students are taller than their parents of the same sex, but accurate figures are difficult to obtain for less selected portions of the population. The children of Japanese origin living in Hawaii and in California, however, are known to be increasing in height at a startling rate. Granting that college students are taller than they used to be, the surprise is that the stretching up has been going on gradually for a century, and that it is not just happening now.

Actually there appear to be some indications that the rate of increase has been slackening in the few generations. The cause of the growth is apparently subtle and credit cannot fairly be assigned to any one factor. The large numbers of tall people that are appearing, no matter why, call for adjustments in architecture and furnishings. For instance chairs should be better designed for long legs, especially in theaters. Automobiles are already making allowances for taller people, but beds are still "standard" in length and Pullman berths none too long.

R. H. O.

Sodium Chlorite.—The properties and reactions of this new commercial chemical were described before the recent meeting of the American Chemical Society by M. C. TAYLOR, J. F. WHITE, G. P. VINCENT, AND G. L. CUNNINGHAM of the Mathieson Alkali Works, New York. The chemical is now being used for bleaching textiles and paper pulp. It is a moderately strong oxidizing agent whose use does not introduce the possibility of heavy metal contamination. The solid sodium chlorite is stable and very soluble. Its solutions are stable in the presence of a trace of alkalinity if light is excluded. They decompose but slowly if boiled. In the storage and handling of chlorite many of the same precautions should be observed as with hypochlorite and chlorate. The oxidizing properties of chlorite are accentuated in acid solution. Such acid solutions of chlorite are employed in bleaching. When acidified in the absence of oxidizable material, chlorite yields mainly chlorine dioxide and chlorate but not chlorine. Chlorite reacts with hypochlorite. In alkaline solutions chlorate is produced. As the alkalinity is reduced some chlorine dioxide is formed. This reaction with hypochlorite can be used to bleach cellulosic materials. In more acid solution the proportionate formation of chlorine dioxide is increased. The addition of chlorine to chlorite solution produces chlorine dioxide very efflciently so that it can serve as a practical source for this useful gas.

R. H. O.

The Amplidyne Generator.—In *General Electric Review*, Vol. 43, No. 3, there appears a series of three papers presented before A. I. E. E. describing the amplidyne generator, its design characteristics, and industrial applications. According to E. F. W. Alexanderson, M. A. Edwards, and K. K. Bowman the apparatus was developed to overcome disadvantages of the high vacuum tube and the thyratron amplifier when applied in power engineering, and to meet new control functions where a high rate of amplification must be combined with a quick and accurate response. It is a dynamoelectric amplifier,

two-stage, making possible amplification of 10,000 to one, and with a much greater amplification in combination with radio amplifiers. Its principal elements are a control field, a short circuit axis on an armature and an output circuit from the armature including a compensating winding. With regard to the principal features of the generator, Alec Fisher states that it is fundamentally an armature-excited machine, in that the armature is essentially the source of main excitation and is also the source of main power output. The first stage of amplification is from a control field to a short circuit axis and the second stage from a short circuit axis to a power-output axis. The total amplification is equal to the product of the first and second stage amplifications. This new type direct current machine has a remarkable range and fidelity of amplification. A number of different uses for the apparatus are described by D. R. Shoults, M. A. Edwards, and F. E. Crever. Among them are the control of machine tools on a multiple head boring mill; a steel strip polisher where such machines consist essentially of unwinding and winding reels, the strip being buffed or polished as it passes between the two reels, and where the drive must maintain substantially constant tension in the strip and should maintain uniform speed; single motor paper machine drive requiring control over a wide range of speeds, using electronic composing amplifier and amplidyne exciter for control of accelerating current and regulation of speed; and an installation of a 5000 h.p. synchronous motor drive for a structural steel mill where the motor excitation is supplied by an amplidyne exciter.

R. H. O.

Kotal—A Product of Industry's Waste Piles.—Vol. 18, No. 6, of *News Edition*, published by the American Chemical Society, states that using industry's waste piles to make new and better products has long been a specialty of chemical science and a new use for manufacturing waste has been developed by Herbert P. Pearson, vice-president of the Asphalt Treatment Corp., New York. Wet stone and gravel and wet road beds have long been a bugaboo of road engineers, for moisture, present when a new asphalt road is laid, will prevent a proper adherence of aggregate and asphalt, the dour result being an imperfect, easily broken travel surface. Standard practice has been to dry all stone and gravel mixes, at a cost of around 25 cents a ton, and to suspend road-building operations in and immediately after wet weather. Pearson's discovery is a liquid, Kotal, made from waste materials, which is applied to stone aggregates, wet or dry, and which affects the asphalt-stone mix in such a way as to give a proper adherence between the two. Mixes prepared in this manner may be laid on wet bottoms. Because stone

and gravel piles are usually wet under the surface and because road building so often has to wait upon the natural drying of the bottom surface before the top course will adhere to it satisfactorily, highway departments have shown interest in Kotal. The final proof will come when the spring thaws have had a chance to attack eighteen experimental road strips in eight different states. Tests by the National Crushed Stone Association on an experimental track paved with Kotalized and untreated asphalt, indicate that "Kotal mixes made with wet stone were superior to the standard Amiesite mixes made with dry stone." Until patents are granted the precise chemical constitution of Kotal will not be revealed, but a cheap waste product was needed in order to compete with the cost of stone drying and today the material cost is found to be 12 to 16 cents per ton of mix.

R. H. O.

New York's Brooklyn-Battery Tunnel.—(*Engineering News-Record*, Vol. 124, No. 12.) Recent arrangements for financing of the twin tube vehicular tunnel in New York City between the Battery in Manhattan to Hamilton Ave. in Brooklyn, have advanced plans for an early start on construction. Each of the tunnels will consist of a circular cast iron shell, lined with concrete, similar to the Holland, Lincoln and Queens-Midtown tunnels. Roadway width will be 21 ft., with annual capacity set at 16,000,000 vehicles. Present traffic through the Holland Tunnel is about 13,500,000. Except for about 200 ft. at the Manhattan end and 1,000 ft. at the Brooklyn end, all of the subaqueous part of the tunnel, some 5,000 ft., will be in rock. Maximum depth below the water surface is 115 ft. at the Governors Island shaft. The Manhattan ventilation building will be made to appear as one of the row of buildings that front on Battery Park. At the Manhattan end traffic will not only be connected to the West Side Elevated Highway, which will be extended 2,500 ft. southward from its present terminus at Duane St., but it will have access, through a tunnel around the rim of Battery Park, to South St., down which it is proposed to extend the East River Drive. The Brooklyn connection from the plaza south to the Belt Parkway at Owls Head Park, nearly 4 mi., will be built by the Triborough Bridge Authority.

R. H. O.

The Preparation of High-Purity Iron on a Large Laboratory Scale.—It was found impossible to supply the needs of the National Physical Laboratory (Great Britain) by the existing apparatus. Accordingly an equipment was provided, a short account of which

is given by Frank Adcock in the *Journal of the Society of Chemical Industry*, Vol. 59, No. 2. It is capable of producing 500 g. of finished iron per working day. Commercial electrolytic iron was converted into ferrous chloride, which was then treated with superheated steam to obtain iron oxide. The finished iron was produced by the hydrogen reduction of the oxide and the subsequent vacuum fusion of the metal. Difficulties were experienced in the provision of suitable refractory crucibles, but these were overcome by the use of sintered alumina.

R. H. O.

Superposed Turbine Blade Research.—(*Combustion*, Vol. 11, No. 9.) F. T. Hague of the Westinghouse Elec. & Mfg. Co. described before the Philadelphia Section A. S. M. E. recently a research study initiated in order to develop a better understanding of the conditions affecting the operation of impulse blades in a superposed turbine. Nothing short of actually photographing the blades' movements under actual operating conditions promised to give adequate and accurate records of the stresses imposed on these blades. To accomplish this an ingenious optical method was devised and the turbine blades are made to write their own story on a camera film adjacent to the turbine. Tiny mirrors are mounted inside eight different blades in the first and second blade rows. A light beam projected through a hole in the turbine shaft is reflected by an inclined mirror to the mirror in the whirling blade being photographed and then back out through the shaft to the stationary camera or viewing screen. By taking pictures every one-half second on a moving film, an exact record of blade performance is obtained for any desired set of operating conditions. Examination of a typical photographic record of a blade's movements during one complete revolution of the spindle shows that the blades vibrate like a tuning fork. The vibration is increased as a blade enters the steam jet, it vibrates while in the jet, and then continues to vibrate with slowly decreasing amplitude after it leaves the jet. This slowly decreasing amplitude of vibration is significant because it is caused by the internal molecular friction of the blade material. The blades vibrate at their natural frequency. This is true whether the steam shock is light or strong and is independent of turbine speed. The natural frequency of vibration is fixed by the blade design and by the material.

R. H. O.